www.mtg.in | March 2018 | Pages 92 | ₹ 30

5 PRACTICE PAPERS 2018

JEE MAIN ADVANCED | WB JEE | BITSAT

MATHEMATICS today

India's #1 MATHEMATICS MONTHLY for JEE (Main & Advanced)

Exam Special

MATH ARCHIVES

MATHS MUSING
10 GREAT PROBLEMS

ACE YOUR WAY
PRACTICE PAPER 2018
CBSE Class XII

MONTHLY PRACTICE PROBLEMS
(XI & XII)

CONCEPT MAP
(XI & XII)

JEE WORK OUTS

You Ask ? We Answer ✓

MTG
Trust of more than 1 Crore Readers Since 1982
2018100010038

MATHEMATICS today

Vol. XXXVI No. 3 March 2018

Corporate Office:
Plot 99, Sector 44 Institutional Area,
Gurgaon -122 003 (HR), Tel : 0124-6601200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.

Managing Editor : Mahabir Singh
Editor : Anil Ahlawat

CONTENTS

Competition Edge

8 Maths Musing Problem Set - 183

10 Practice Paper - JEE Advanced

15 JEE Work Outs

18 Practice Paper - BITSAT

27 Math Archives

30 Gear Up for JEE Main

36 Practice Paper - JEE Main

45 You Ask We Answer

48 Mock Test Paper - JEE Main

56 Practice Paper - JEE Main

63 Practice Paper - WBJEE

72 Maths Musing Solutions

Class XI

46 Concept Map

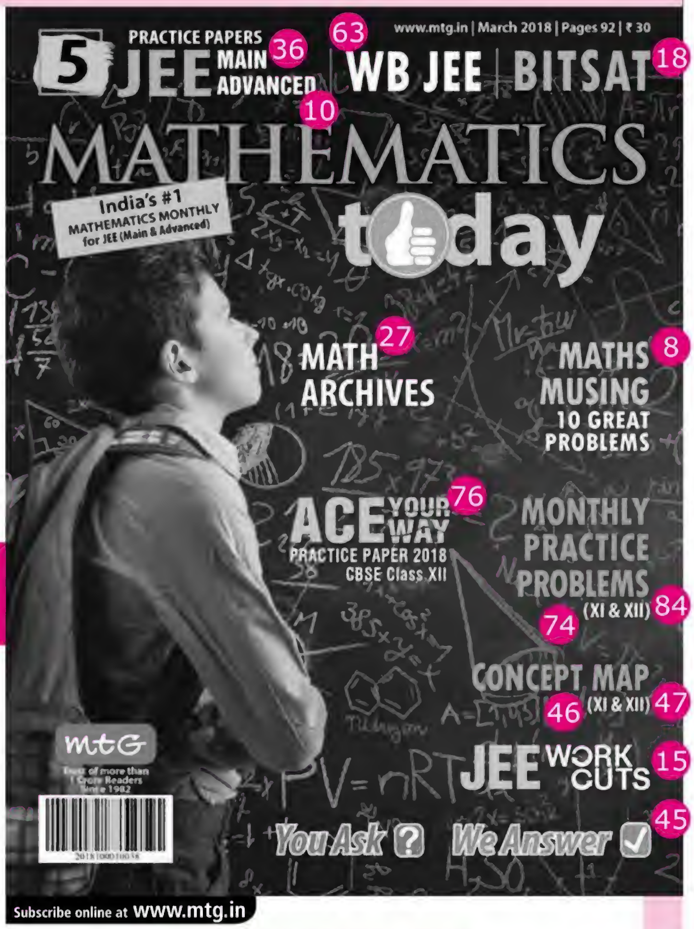
74 MPP

Class XII

47 Concept Map

76 Ace Your Way Practice Paper

84 MPP



Printed and Published by Mahabir Singh on behalf of MTG Learning Media Pvt. Ltd. Printed at HT Media Ltd., B-2, Sector-63, Noida, UP-201307 and published at 406, Taj Apartment, Ring Road, Near Safdarjung Hospital, New Delhi - 110029.
Editor : Anil Ahlawat

MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.
During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.
Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 183

JEE MAIN

1. The value of $\int_1^a [x]\, f'(x)\, dx,\ a > 1$, where $[x]$ denotes the greatest integer not exceeding x is
(a) $[a]f(a) - \{f(1) + f(2) + ... + f([a])\}$
(b) $[a]f([a]) - \{f(1) + f(2) + ... + f(a)\}$
(c) $a\, f([a]) - \{f(1) + f(2) + ... + f(a)\}$
(d) $a\, f(a) - \{f(1) + f(2) + ... + f([a])\}$

2. In a binomial distribution $B\left(n, p = \frac{1}{4}\right)$, if the probability of atleast one success is greater than or equal to 9/10, then $n >$
(a) $\frac{1}{\log_{10} 4 - \log_{10} 3}$ (b) $\frac{1}{\log_{10} 4 + \log_{10} 3}$
(c) $\frac{9}{\log_{10} 4 - \log_{10} 3}$ (d) $\frac{4}{\log_{10} 4 - \log_{10} 3}$

3. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P with respect to additional number of workers x is given by $dP/dx = 100 - 12\sqrt{x}$. If the firm employees 25 more workers, then the new level of production of items is
(a) 2500 (b) 3000 (c) 3500 (d) 4500

4. If $f(x) = \cos x - \cos^2 x + \cos^3 x - ... \infty$, then $\int f(x)\, dx =$
(a) $\tan\frac{x}{2} + C$ (b) $x - \tan\frac{x}{2} + C$
(c) $x - \frac{1}{2}\tan\frac{x}{2} + C$ (d) $\frac{x - \tan x/2}{2} + C$

5. If the vector $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a ΔABC, then the length of the median through A is
(a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{45}$

JEE ADVANCED

6. Consider the family of all circles whose centres lie on the straight line $y = x$. If this family of circles is represented by the differential equation $Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and y', then which of the following statements is (are) true?
(a) $P = y + x$ (b) $P = y - x$
(c) $P + Q = 1 - x + y + y' + (y')^2$
(d) $P - Q = x + y - y' - (y')^2$

COMPREHENSION

Two lines whose equations are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ lie in the same plane.

7. The value of $\sin^{-1}(\sin\lambda)$ is equal to
(a) 3 (b) $\pi - 3$ (c) 4 (d) $\pi - 4$

8. Point of intersection of the lines lies on
(a) $3x + y + z = 20$ (b) $2x + y + z = 25$
(c) $3x + 2y + z = 24$ (d) $x = y = z$

INTEGER MATCH

9. The value of $e^{2mi\cot^{-1}p}\cdot\left(\frac{pi+1}{pi-1}\right)^m$ is

MATRIX MATCH

10. Match the following.

	List-I	List-II	
P.	The number of integral solutions of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is	1.	5
Q.	The number of real values of x satisfying $1 + \lvert e^x - 1\rvert = e^x(e^x - 2)$	2.	4
R.	The number of integral solutions of $\lvert [x] - 2x\rvert = 4$ where $[\cdot]$ denotes G.I.F is	3.	1
S.	The number of integral solutions satisfying $\lvert\lvert x-1\rvert - 1\rvert \le 1$	4.	3

	P	Q	R	S
(a)	1	2	3	4
(b)	4	3	2	1
(c)	2	3	1	4
(d)	3	2	4	1

See Solution Set of Maths Musing 182 on page no 72

JEE ADVANCED

PRACTICE PAPER 2018

Exam on 20th May

SECTION-1

SINGLE CORRECT ANSWER TYPE

1. Let $2(1+x^3)^{100} = \sum_{i=0}^{100}\left(a_i x^i - \cos\frac{\pi(x+i)}{2}\right)$ then $\sum_{i=0}^{50} a_{2i} =$

(a) 2^{100} (b) 2^{99} (c) 2^{50} (d) 2^{49}

2. From any point on the circle $x^2 + y^2 = 9$, two tangents are drawn to circle $x^2 + y^2 = 4$ which cut $x^2 + y^2 = 9$ at A and B then locus of point of intersection of tangents drawn to $x^2 + y^2 = 9$ at A and B is

(a) $x^2 + y^2 = 27$ (b) $x^2 + y = 27$
(c) $x^2 + y^2 = 27^2$ (d) $x^2 + y = 27^2$

3. Let A be a 2×2 matrix with real entries and $\det(A) = t \neq 0$ such that $\det(A + t(\text{adj } A)) = 0$ then $\det(A - t(\text{adj } A)) =$

(a) 0 (b) 2 (c) 4 (d) 8

4. Let $A(\omega^2)$, $B(2i\omega)$, $C(-4)$ be three points lying on the complex plane. On the circumcircle of ΔABC, a point P is taken such that $PA \cdot BC = PC \cdot AB$ (where P, A, B, C are in order). If z is the affix of midpoint of PB then $|z|^2 =$

(a) 0 (b) 1 (c) 2 (d) 3

5. Let 'a' and 'b' be distinct, randomly chosen roots of the equation $z^{210} - 1 = 0$. The probability that $|a+b| \geq \sqrt{2+\sqrt{3}}$ is (approx.)

(a) 0.16 (b) 0.22 (c) 0.31 (d) 0.54

6. A bag contains 21 red and 21 black balls. We remove 2 balls at a time repeatedly and discard them if they are of the same colour, but if they are different then we discard the black ball and return the red ball. The probability that this process will terminate with 1 red ball in the bag is

(a) 0 (b) 1/2 (c) 1/3 (d) 1

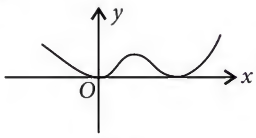
7. $\int_0^{\infty} (\sqrt{x^2+1} + x)^{-3}\, dx =$

(a) 1/8 (b) 2/8 (c) 3/8 (d) 4/8

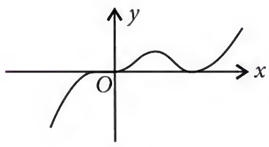
SECTION-2

MORE THAN ONE CORRECT ANSWER TYPE

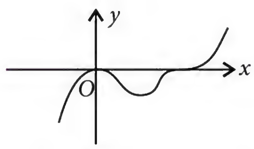
8. Let $f(x) = x^m(x-1)^n$, $m, n \in I$ and $(m, n > 1)$ then possible graph of $f(x)$ in various cases of m and n is/are

(a) 

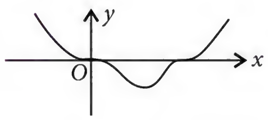
m even
n even

(b) 

m odd
n even

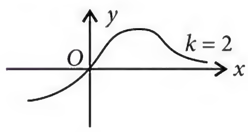
(c) 

m even
n odd

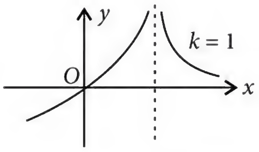
(d) 

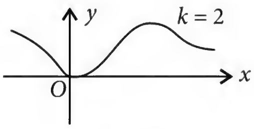
m odd
n odd

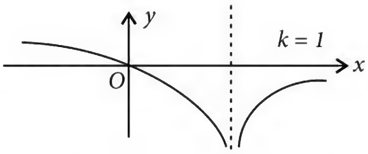
9. The possible graph of $f(x) = \dfrac{x}{\sqrt{x^2 - 2x + k}}$ is/are

(a) 

By : Tapas Kr. Yogi, Mob : 9533632105.

(b) 

(c) 

(d) 

10. A damped system with feedback is given by the differential equation $f'(t) = -f(t) + kf(t-1)$ where k is a given non-zero constant. The non-zero solutions of f of the form $f(t) = ae^{mt}$, where 'a' and 'm' are constants, are possible provided m satisfies the equation $g(m) = 0$. Then
(a) $g(m) = (m+1) - ke^{-m}$
(b) $g(m) = (m-1) + ke^{m}$
(c) for $k > 0$, $g(m) = 0$ has no solution in m
(d) for $k > 0$, $g(m) = 0$ has exactly one solution in m

11. The possible value of the expression
$$\frac{\sec^4\alpha}{\tan^2\beta}+\frac{\sec^4\beta}{\tan^2\alpha}\quad\left(\alpha,\beta\neq\frac{n\pi}{2},\ n\in I\right)\text{ is/are}$$
(a) 6 (b) 8 (c) 10 (d) 12

12. Let $\theta = \pi/7$ then
(a) $\cos\theta$ is not a root of the equation $8x^3 + 4x^2 - 4x - 1 = 0$
(b) $\cos\theta$ is irrational
(c) $\cos\theta-\cos 2\theta+\cos 3\theta=\frac{1}{2}$
(d) $\sin^2 3\theta-\sin^2\theta=\sin 2\theta\sin 3\theta$

13. A straight line through $P(\lambda, 2)$, $(\lambda \neq 0)$ meets the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A and D meets the axes at B and C such that PA, PB, PC, PD are in G.P. then $\lambda \in$
(a) $[7, \infty)$ (b) $(-12, -8)$
(c) $(-5, 0)$ (d) $(10, \infty)$

14. Let $P(x) = x^3 + x + 1$ and $Q(x)$ be a polynomial such that the roots of $Q(x) = 0$ are the squares of the roots of $P(x) = 0$ then
(a) $Q(4)$ is divisible by 11
(b) the last two digits of $(Q(1))^{50}$ is 29
(c) when $(Q(1))^{50}$ is divided by 5, then remainder is 4
(d) the probability that $Q(n)$ is an odd integer is 1; $(n \in N)$

SECTION-3
COMPREHENSION TYPE

Passage-1

A rectangle $ABCD$ of dimensions r and $2r$ is folded along diagonal BD such that planes ABD and CBD are normal to each other. Let the position of vertex A remain unchanged and C_1 is the new position of C.

15. The distance of C_1 from A is equal to
(a) $\frac{\sqrt{85}r}{5}$ (b) $\frac{\sqrt{75}r}{5}$ (c) $\frac{\sqrt{65}r}{5}$ (d) $\frac{\sqrt{55}r}{5}$

16. If $\angle ABC_1 = \alpha$ then $\cos\alpha =$
(a) $\frac{1}{5}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{1}{10}$ (d) $\frac{1}{\sqrt{10}}$

Passage-2

In a cosmological model, the radius R of the universe is a function of the age t of the universe. The function R satisfies the following three conditions
(i) $R(0) = 0$ (ii) $R'(t) > 0$ for $t > 0$ and
(iii) $R''(t) < 0$ for $t > 0$

A function $H(t)$ is defined as $H_{(t)}=\frac{R'(t)}{R(t)}$

17. By the graph of $R(t)$ we see that
(a) $t<\frac{1}{H_{(t)}}$ (b) $t>\frac{1}{H_{(t)}}$
(c) $t<\frac{2}{H_{(t)}}$ (d) $t>\frac{2}{H_{(t)}}$

18. If $H_{(t)}=\frac{a}{t}$ where a is a constant then $a \in$
(a) $(0, 1)$ (b) $(1, 2)$
(c) $(0, 1/2)$ (d) $(1/2, 1)$

SOLUTIONS

1. **(a) :** Given $2(1+x^3)^{100} = (a_0 + a_1x + ... + a_{100}x^{100})$
$$-\left(\cos\frac{\pi x}{2}+\cos\frac{\pi(x+1)}{2}+.....+\cos\frac{\pi}{2}(x+100)\right)$$
Putting $x = 1$ and $x = -1$ alternately and then adding, we have $\sum_{i=0}^{50} a_{2i} = 2^{100}$

2. **(c) :** Let R be the point of intersection of the tangents at A and B then from ΔOAP, $\sin\theta = 2/3$ where $\angle OAP = \theta$ and so $\angle AOR = 2\theta$ gives $\cos 2\theta = \frac{3}{OR}$.

Solving, we have $OR = 27$

i.e. R is at a fixed distance from O.

Hence, locus is circle $\sqrt{x^2+y^2} = 27$.

3. **(c) :** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $ad - bc = t$ and

$\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ and

$|A + t\,\text{adj}(A)| = 0 \Rightarrow \begin{vmatrix} a+td & b-bt \\ c-ct & d+at \end{vmatrix} = 0$

On simplification, we get $(a + d)^2 + (t - 1)^2 = 0$

i.e. $a + d = 0$ and $t = 1$

and $|A - t(\text{adj}.A)| = t(1 + t)^2 = 4$

4. **(a) :** Let z_p be the affix of point P then applying Rotation theorem at P and B, we get

$$\frac{z_p+4}{z_p-\omega^2}\cdot\frac{2i\omega-\omega^2}{2i\omega+4} = -1$$

Simplifying, we have $z_p = -2i\omega$

So, $z = \frac{z_p+2i\omega}{2} = 0$ *i.e.*, $|z|^2 = 0$

5. **(a) :** As all the 210 roots of the equation are symmetrically distributed in the complex plane, we can assume $a = 1$ without loosing generality. So, the given condition becomes

$|1 + b|^2 = |1 + \cos\theta + i\sin\theta|^2 = 2 + 2\cos\theta \geq 2+\sqrt{3}$ (ATQ)

i.e. $\cos\theta \geq \frac{\sqrt{3}}{2}$ *i.e.*, $|\theta| \leq \frac{\pi}{6}$

As $b \neq 1$, θ is of the form $\pm\frac{2k\pi}{210}$ where

$k \in \left[1, \left[\frac{210}{12}\right]\right] = [1, 17]$

There are $2 \times 17 = 34$ such angles.

So, the required probability $= \frac{34}{209} \approx 0.16$

6. **(d) :** As atleast one ball is removed from each stage, the process will eventually end with either no ball or one ball. Because red balls are odd in number at start and we are removing 2 at a time, the number of red balls at any time is odd. Hence, the process will always leave red balls in the bag. So, it ends with exactly 1 red ball. Hence, required probability = 1.

7. **(c) :** Let $I = \int_0^\infty (\sqrt{x^2+1}+x)^{-3}\,dx$

Put $x + \sqrt{x^2+1} = t$ *i.e.* $x = \frac{t^2-1}{2t}$

$\therefore \; I = \frac{1}{2}\int_1^\infty (t^{-3}+t^{-5})\,dt = \frac{3}{8}$

8. **(a, b, c, d) :** $f'(x) = f(x)\cdot\left(\frac{m}{x}-\frac{n}{1-x}\right)$

and $f''(x) = f(x)\left(\frac{-m}{x^2}-\frac{n}{(x-1)^2}\right) + f(x)\left(\frac{m}{x}+\frac{n}{x-1}\right)^2$

If $f(x) > 0$ then $f''(x) < 0$ and if $f(x) < 0$, then $f''(x) > 0$.

9. **(a, b) :** $\frac{dy}{dx} = \frac{(k-x)}{(x^2-2x+k)^{3/2}}$

When $k = 2$, the stationary point is at $(2, \sqrt{2})$ and $x = 0$, $y = 0$; $x \to \infty$, $y \to 1$; $x \to -\infty$, $y \to -1$

When $k = 1$, then $y = \frac{x}{|x-1|}$

vertical asymptote at $x = 1$.

10. **(a, d) :** Putting $f(t) = ae^{mt}$ in the given differential equation, we have

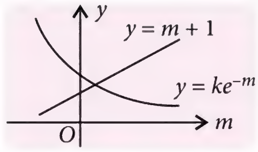
$m\cdot e^{mt} = -e^{mt} + k\cdot e^{m(t-1)}$

i.e. $m + 1 = ke^{-m}$

Hence, $g(m) = (m + 1) - ke^{-m}$

When $k > 0$, $g(m) = 0$

i.e., $(m + 1) = ke^{-m}$



Graphs intersect at only one point.

11. **(b, c, d) :** Let $\tan^2\alpha = a$, $\tan^2\beta = b$, then the given expression reduces to

$$\frac{(a+1)^2}{b}+\frac{(b+1)^2}{a} = 2\left(\frac{a}{b}+\frac{b}{a}\right)+\left(\frac{a^2}{b}+\frac{b^2}{a}+\frac{1}{a}+\frac{1}{b}\right)$$

$$\geq 2(2) + 4(1) = 8$$

Hence, minimum value = 8 at $a = b = 1$

i.e. $\alpha = n\pi \pm \frac{\pi}{4}$, $\beta = n\pi \pm \frac{\pi}{4}$

12. **(a, b, c, d) :** Let $\theta = \frac{\pi}{7}$, $7\theta = \pi \rightarrow 4\theta = \pi - 3\theta$

So, $\sin 4\theta = \sin 3\theta$

i.e., $2\sin 2\theta \cos 2\theta = 3\sin\theta - 4\sin^3\theta$

Simplifying further, we get $8\cos^3\theta - 4\cos^2\theta - 4\cos\theta + 1 = 0$

So $(2\cos\theta)$ is root of the equation $t^3 - t^2 - 2t + 1 = 0$
Now, we see that $t = 1$ or $t = -1$ are not the roots.
So, $(2\cos\theta)$ is irrational $\Rightarrow$ $\cos\theta$ is irrational.

13. (a, b, d) : Any point on the cone through $P(\lambda, 2)$ is $(\lambda + r\cos\theta, 2 + r\sin\theta)$ and meets the ellipse at A and D

So, $\dfrac{(\lambda + r\cos\theta)^2}{9} + \dfrac{(2 + r\sin\theta)^2}{4} = 1$

So, $r_1 r_2 = PA \cdot PD = \dfrac{4\lambda^2}{4\cos^2\theta + 9\sin^2\theta}$

Again solving with $x = 0$ and $y = 0$, we have

$PB \cdot PC = \dfrac{2\lambda}{\sin\theta\cos\theta}$

So, $\dfrac{4\lambda^2}{4\cos^2\theta + 9\sin^2\theta} = \dfrac{2\lambda}{\sin\theta\cos\theta}$

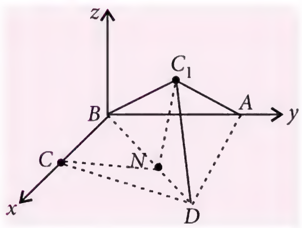
i.e., $2\lambda\sin 2\theta + 5\cos 2\theta = 13$

So, $-1 \le \dfrac{13}{\sqrt{4\lambda^2 + 25}} \le 1$ *i.e.*, $|\lambda| \ge 6$

14. (a, c, d) : Let $x = \alpha^2$, $\alpha = \sqrt{x}$ put in $P(x) = 0$, we get $Q(x) = x^3 + 2x^2 + x - 1$.

(15 – 16) :

15. (a) **16. (a)**



Let rectangle $ABCD$ be on xy plane.
and equation of BD is $y = 2x$
$A(0, 2r, 0)$, $D(r, 2r, 0)$, $C(r, 0, 0)$

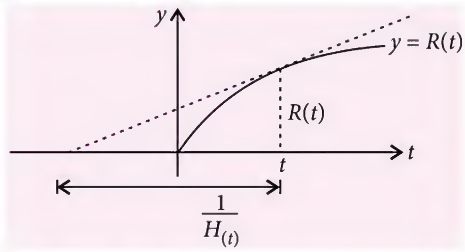
Foot of perpendicular N on $BD = \left(\dfrac{r}{5}, \dfrac{2r}{5}, 0\right)$

$CN = C_1N \Rightarrow C_1 \equiv \left[\dfrac{r}{5}, \dfrac{2r}{5}, \dfrac{2r}{\sqrt{5}}\right]$

$\therefore\ C_1A = \dfrac{\sqrt{85}}{5} r$ and $\cos\alpha = \dfrac{1}{5}$

(17 – 18) :

17. (a) **18. (a)**



$\dfrac{1}{H_{(t)}}$ is the length of the base of the right triangle with height $R(t)$.

So, from the graph it is clear that $\dfrac{1}{H_{(t)}} > t$

Now, for $H_{(t)} = \dfrac{a}{t} = \dfrac{R'(t)}{R(t)} \Rightarrow R(t) = c \cdot t^a$

and if $a > 0$, $c > 0$ and $R''(t) = c \cdot a(a - 1)t^{a-2} < 0$

$\Rightarrow\ a < 1$

So, $a \in (0, 1)$

JEE WORK OUTS

1. Let (x_0, y_0) be the solution of the following equations $(2x)^{\ln 2} = (3y)^{\ln 3}$ and $3^{\ln x} = 2^{\ln y}$. Then x_0 is
   (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 6

2. Let $S(K) = 1 + 3 + 5 + \ldots\ldots + (2K - 1) = 3 + K^2$. Then which of the following is true ?
   (a) $S(1)$ is correct
   (b) Principle of mathematical induction can be used to prove the formula
   (c) $S(K) \nRightarrow S(K + 1)$
   (d) $S(K) \Rightarrow S(K + 1)$

3. The real part of $(1 - \cos\theta + 2i \sin\theta)^{-1}$ is
   (a) $\frac{1}{3+5\cos\theta}$ (b) $\frac{1}{5-3\cos\theta}$
   (c) $\frac{1}{3-5\cos\theta}$ (d) $\frac{1}{5+3\cos\theta}$

4. a, g, h are arithmetic mean, geometric mean and harmonic mean between two positive numbers x and y respectively. Then identify the correct statement among the following.
   (a) h is the harmonic mean between a and g
   (b) No such relation exists between a, g and h
   (c) g is the geometric mean between a and h
   (d) a is the arithmetic mean between g and h

5. If $s_r = \alpha^r + \beta^r + \gamma^r$, then the value of $\begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix}$ is equal to
   (a) 0 (b) $(\alpha - \beta)(\beta - \gamma)(\gamma - \alpha)$
   (c) $(\alpha + \beta + \gamma)^6$ (d) $(\alpha - \beta)^2 (\beta - \gamma)^2 (\gamma - \alpha)^2$

6. p : Sita is beautiful; q = Sita is clever.
   When $\sim p \vee q$ is written in verbal form it becomes
   (a) Sita is not beautiful or Sita is not clever
   (b) Sita is beautiful and she is clever.
   (c) Sita is not beautiful or Sita is clever
   (d) Sita is beautiful and Sita is not clever

7. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is
   (a) $\frac{10!}{6}$ (b) 3! 7!
   (c) ${}^{10}P_3 \cdot 7!$ (d) None of these

8. If the coefficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$, then ab =
   (a) 1 (b) 1/2 (c) 2 (d) 3

9. If the probability density function of a random variable X is $f(x) = x/2$ in $0 \le x \le 2$, then $P(X > 1.5 \mid X > 1)$ is equal to
   (a) 7/16 (b) 3/4 (c) 7/12 (d) 21/64

10. The product of lengths of perpendiculars drawn from the origin to the lines represented by the equation,
    $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, will be
    (a) $\frac{ab}{\sqrt{a^2 - b^2 + 4h^2}}$ (b) $\frac{bc}{\sqrt{a^2 - b^2 + 4h^2}}$
    (c) $\frac{ca}{\sqrt{(a^2 + b^2) + 4h^2}}$ (d) $\frac{c}{\sqrt{(a-b)^2 + 4h^2}}$

By : **Vidyalankar Institute**, Pearl Centre, Senapati Bapat Marg, Dadar (W), Mumbai - 28. Tel.: (022) 24306367

11. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, (where $pq \neq 0$) are bisected by the x-axis, then

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$
(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

12. A circle C_1 of radius 2 touches both x-axis and y-axis. Another circle C_2 whose radius is greater than 2 touches circle C_1 and both the axes. Then the radius of circle C_2 is

(a) $6-4\sqrt{2}$ (b) $6+4\sqrt{2}$
(c) $6-4\sqrt{3}$ (d) $6+4\sqrt{3}$

13. If $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then $\sin\alpha + \cos\alpha$ and $\sin\alpha - \cos\alpha$ must be respectively equal to

(a) $\sqrt{2}\cos\theta, \sqrt{2}\sin\theta$ (b) $\sqrt{2}\sin\theta, \sqrt{2}\cos\theta$
(c) $\sqrt{2}\sin\theta, \sqrt{2}\sin\theta$ (d) $\sqrt{2}\cos\theta, \sqrt{2}\cos\theta$

14. A pair of straight lines drawn through the origin form with the line $2x + 3y = 6$ an isosceles right angled triangle, then the area of the triangle thus formed is

(a) $\Delta = \frac{36}{13}$ (b) $\Delta = \frac{12}{17}$
(c) $\Delta = \frac{13}{5}$ (d) None of these

15. Point D, E are taken on the side BC of a triangle ABC such that $BD = DE = EC$. If $\angle BAD = x$, $\angle DAE = y$, $\angle EAC = z$, then the value of $\frac{\sin(x+y)\sin(y+z)}{\sin x \sin z} =$

(a) 1 (b) 2
(c) 4 (d) None of these

16. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $x + y + z$ is equal to

(a) xyz (b) 0 (c) 1 (d) $2xyz$

17. From the bottom of a pole of height h, the angle of elevation of the top of a tower is α and the pole subtends angle β at the top of the tower. The height of the tower is

(a) $\frac{h\tan(\alpha-\beta)}{\tan(\alpha-\beta)-\tan\alpha}$ (b) $\frac{h\cot(\alpha-\beta)}{\cot(\alpha-\beta)-\cot\alpha}$
(c) $\frac{\cot(\alpha-\beta)}{\cot(\alpha-\beta)-\cot\alpha}$ (d) None of these

18. If f is an even function defined on the interval $(-5, 5)$, then four real values of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$ are

(a) $\frac{-3-\sqrt{5}}{2}, \frac{-3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}$
(b) $\frac{-5+\sqrt{3}}{2}, \frac{-3+\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$
(c) $\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}, \frac{-3-\sqrt{5}}{2}, \frac{5+\sqrt{3}}{2}$
(d) $-3-\sqrt{5}, -3+\sqrt{5}, 3-\sqrt{5}, 3+\sqrt{5}$

19. The value of $\lim_{x\to\frac{\pi}{2}} \frac{\int_{\pi/2}^{x} t\,dt}{\sin(2x-\pi)}$ is

(a) ∞ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

20. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$ then $f^2(4) + g^2(4)$ is

(a) 8 (b) 16 (c) 32 (d) 64

21. The value of p for which the function,

$$f(x) = \begin{cases} \dfrac{(4^x-1)^3}{\sin\dfrac{x}{p}\log_e\left(1+\dfrac{x^2}{3}\right)}, & x \neq 0 \\ 12(\log_e 4)^3, & x = 0 \end{cases}$$

is continuous at $x = 0$, is

(a) 1 (b) 2
(c) 3 (d) None of these

22. Maximize $z = 3x + y$ subject to $x + y \leq 6$, $y \leq 4$, $x \geq 1$, $x \geq 0$ and $y \geq 0$.

(a) 20 (b) 18 (c) 10 (d) 15

23. Let $f(x) = \int_0^x \frac{\cos t}{t}dt, x > 0$ then $f(x)$ has

(a) Maxima when $n = -2, -4, -6, \ldots.$
(b) Maxima when $n = -1, -3, -5, \ldots.$
(c) Maxima when $n = 1, 3, 5, \ldots.$
(d) None of these

MPP CLASS XI ANSWER KEY

1. (b)	2. (d)	3. (d)	4. (d)	5. (a)
6. (c)	7. (a,b)	8. (b,d)	9. (a)	10. (b,c)
11. (a,c)	12. (a,b)	13. (a,b,c)	14. (d)	15. (a)
16. (d)	17. (6)	18. (1)	19. (0)	20. (2)

24. Let, P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral $PQRS$ must be a
(a) Parallelogram, which is neither a rhombus nor a rectangle
(b) Square
(c) Rectangle, but not a square
(d) Rhombus, but not a square

25. Let $A = \{2, 3, 4, 5\}$ and let $R\{(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)\}$ be a relation in A. Then R is
(a) reflexive and transitive
(b) reflexive and symmetric
(c) transitive and symmetric
(d) equivalence

26. The value of 'a' so that the volume of parallelepiped formed by $\hat{i}+a\hat{j}+a\hat{k}, \hat{j}+a\hat{k}$ and $a\hat{i}+\hat{k}$ becomes minimum, is
(a) -3 (b) 3 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

27. $\int \frac{dx}{(1+x^2)\sqrt{p^2+q^2(\tan^{-1}x)^2}} =$
(a) $\frac{1}{q}\log\left[q\tan^{-1}x+\sqrt{p^2+q^2(\tan^{-1}x)^2}\right]+c$
(b) $\log\left[q\tan^{-1}x+\sqrt{p^2+q^2(\tan^{-1}x)^2}\right]+c$
(c) $\frac{2}{3q}(p^2+q^2\tan^{-1}x)^{3/2}+c$
(d) None of these

28. Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is :
(a) 80 (b) 50 (c) $82\frac{1}{2}$ (d) 48

29. A tangent to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the co-ordinate axes in P and Q. The least value of PQ is
(a) $a-b$ (b) $a+b$ (c) $\frac{a+b}{2}$ (d) $2a$

30. The plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, 1, 1)$ and $(1, -1, 2)$ makes intercepts on the co-ordinate axes, the sum of length of intercepts is
(a) 3 (b) 4 (c) 6 (d) 12

ANSWER KEY

1.	(c)	2.	(d)	3.	(d)	4.	(c)
5.	(d)	6.	(c)	7.	(a)	8.	(a)
9.	(c)	10.	(d)	11.	(d)	12.	(b)
13.	(a)	14.	(a)	15.	(c)	16.	(a)
17.	(b)	18.	(a)	19.	(c)	20.	(c)
21.	(d)	22.	(b)	23.	(b)	24.	(a)
25.	(b)	26.	(c)	27.	(a)	28.	(b)
29.	(b)	30.	(d)				

For detailed solution to the Sample Paper, visit our website www. vidyalankar.org

PRACTICE PAPER

BITSAT

Exam date : 16th to 31st May 2018

1. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle through P, Q and $(1, 1)$ for
   (a) all values of p
   (b) all except one value of p
   (c) all except two values of p
   (d) exactly one value of p

2. Any point on the parabola with focus $(0, 1)$ and directrix $x + 2 = 0$ is
   (a) $(t^2 + 1, 2t - 1)$ (b) $(t^2 + 1, 2t + 1)$
   (c) $(t^2, 2t)$ (d) $(t^2 - 1, 2t + 1)$

3. The sum to n terms of the series
   $$\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots\ldots \text{ is equal to}$$
   (a) $\frac{3n}{n+1}$ (b) $\frac{6n}{n+1}$ (c) $\frac{9n}{n+1}$ (d) $\frac{12n}{n+1}$

4. A pair of tangents are drawn from the point $(-1, 1)$ to $x^2 - 4x + 4y^2 = 0$. If θ is the angle between them, then $\tan\theta$ is
   (a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{6}{5}$ (d) $\frac{3\sqrt{3}}{5}$

5. The distance between the vertex of the parabola $y = x^2 - 4x + 3$ and centre of the circle $x^2 = 9 - (y - 3)^2$ is
   (a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $2\sqrt{2}$ (d) $2\sqrt{5}$

6. Given two numbers a and b. Let A denote the single A.M. and S denote the sum of n A.M.'s between a and b, then S/A depends on
   (a) n, a, b (b) n, b (c) n, a (d) n

7. If $\cos^{-1}\frac{y}{b} = n\log\frac{x}{n}$, then $x^2y_2 =$
   (a) $xy_1 + n^2y$ (b) $xy_1 - n^2y$
   (c) $-xy_1 + n^2y$ (d) $-xy_1 - n^2y$

8. $$\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3}+\frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3}+\frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+3^3}+\ldots \text{ to } n \text{ terms} =$$
   (a) $\frac{n^2}{(n+1)^2}$ (b) $\frac{n^3}{(n+1)^3}$
   (c) $\frac{n}{n+1}$ (d) $\frac{1}{n+1}$

9. The number of roots of
   $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{2}$ is
   (a) 0 (b) 1 (c) 2 (d) > 2

10. The value of the integral $\sum_{k=1}^{n}\int_0^1 f(k-1+x)dx$ is
    (a) $\int_0^1 f(x)dx$ (b) $\int_0^2 f(x)dx$
    (c) $\int_0^n f(x)dx$ (d) $n\int_0^1 f(x)dx$

11. A chord of the hyperbola $4x^2 - 9y^2 = 36$ is bisected at the point $(3, 5)$. The distance of the origin from the chord is
    (a) $\frac{63}{\sqrt{241}}$ (b) $\frac{50}{\sqrt{241}}$ (c) $\frac{13}{\sqrt{19}}$ (d) $\frac{13}{\sqrt{190}}$

12. In a triangle ABC, if $a = 3$, $b = 4$, $c = 5$, then $r_1 + r_2 + r_3 =$
    (a) 9 (b) 10 (c) 11 (d) 12

13. $\int x^n \log x\,dx =$
    (a) $\frac{x^{n+1}}{n}[n\log x - 1] + C$
    (b) $\frac{x^{n+1}}{n+1}[(n+1)\log x - 1] + C$
    (c) $\frac{x^{n+1}}{(n+1)^2}[(n+1)\log x - 1] + C$
    (d) none of these

14. The differential equation $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circles with
(a) variable radii and a fixed centre (0, 1)
(b) variable radii and fixed centre (0, –1)
(c) fixed radius 1 and variable centres along the x-axis
(d) fixed radius 1 and variable centres along the y-axis

15. If p : 4 is an even prime number,
q : 6 is a divisor of 12 and
r : the HCF of 4 and 6 is 2.
Which one of the following is true ?
(a) $(p \wedge q)$ (b) $(p \vee q) \wedge \sim r$
(c) $\sim (q \wedge r) \vee p$ (d) $\sim p \vee (q \wedge r)$

16. $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}} =$
(a) $\frac{(x^4+1)^{\frac{1}{4}}}{x}+c$ (b) $-\frac{(x^4+1)^{\frac{1}{4}}}{x}+c$
(c) $\frac{\sqrt{x^4+1}}{x}+c$ (d) $-\frac{\sqrt{x^4+1}}{x}+c$

17. If $f(x)=\begin{cases}\int_0^x (1+|1-t|)\,dt, & x>2\\ 5x-7, & x\le 2\end{cases}$, then
(a) $f(x)$ is not continuous at $x = 2$
(b) f is differentiable everywhere
(c) R.H.L. at $x = 2$ doesn't exist
(d) f is continuous but not differentiable at $x = 2$

18. $x_1, x_2, x_3, \ldots, x_{50}$ are fifty real numbers such that $x_r < x_{r+1}$ for $r = 1, 2, 3, \ldots, 49$. Five numbers out of these are picked up at random. The probability that the five numbers have x_{20} as the middle number is
(a) $\frac{^{20}C_2 \times {}^{30}C_2}{^{50}C_5}$ (b) $\frac{^{30}C_2 \times {}^{19}C_2}{^{50}C_5}$
(c) $\frac{^{19}C_2 \times {}^{31}C_2}{^{50}C_5}$ (d) none of these

19. Let $f(x) = x^3 \operatorname{sgn} x$ for all $x \in R$, then
(a) f is continuous but not derivable at 0
(b) f is derivable at 0
(c) $Lf'(0) = -3$ (d) $Rf'(0) = 3$

20. The incentre of the triangle formed by the coordinates axes and $3x + 4y = 12$ is
(a) (1/2, 1/2) (b) (1, 1)
(c) (1, 1/2) (d) (1/2, 1)

21. In a triangle ABC, if $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c} = \frac{a}{bc}+\frac{b}{ca}$, then $A =$
(a) 30° (b) 45° (c) 60° (d) 90°

22. If $\bar{X}_1$ and $\bar{X}_2$ are the means of two distributions such that $\bar{X}_1 < \bar{X}_2$ and $\bar{X}$ is the mean of the combined distribution, then
(a) $\bar{X} < \bar{X}_1$ (b) $\bar{X} > \bar{X}_2$
(c) $\bar{X} = \frac{\bar{X}_1+\bar{X}_2}{2}$ (d) $\bar{X}_1 < \bar{X} < \bar{X}_2$

23. $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots\infty}}}$, then $\frac{dy}{dx} =$
(a) $\frac{2y-1}{\cos x}$ (b) $\frac{\cos x}{2y-1}$ (c) $\frac{2x-1}{\cos y}$ (d) $\frac{\cos y}{2y-1}$

24. The number of values of k for which the equation $x^2 - 3x + k = 0$ has two distinct roots lying in the interval (0, 1) are
(a) three
(b) two
(c) infinitely many
(d) no value of k satisfies the requirement

25. If $S=\sum_{n=2}^{\infty}\frac{^nC_2}{(n+1)!}$, then S equals
(a) $e - 2$ (b) $e + 2$ (c) $2e$ (d) $\frac{e}{2}-1$

26. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ and $f(1) = 2$, $f(p + q) = f(p) \cdot f(q)$, $\forall\, p, q \in R$, then
$x^{f(1)}+y^{f(2)}+z^{f(3)}-\frac{(x+y+z)}{x^{f(1)}+y^{f(2)}+z^{f(3)}}$
is equal to
(a) 0 (b) 1 (c) 2 (d) 3

27. Area between the curves $y = x^3$ and $y=\sqrt{x}$ is
(a) $\frac{5}{12}$ (b) $\frac{5}{3}$
(c) $\frac{5}{4}$ (d) none of these

28. $\int_0^{\pi/2} \frac{(\sin x+\cos x)^2}{\sqrt{1+\sin 2x}} dx =$

(a) 0 (b) 1 (c) 2 (d) 3

29. If the line $y = x\sqrt{3}$ cuts the curve $x^3 + y^3 + 3xy + 5x^2 + 3y^2 + 4x + 5y - 1 = 0$ at the points A, B and C, then $OA \cdot OB \cdot OC$ (where O is (0, 0)) is

(a) $\frac{4}{13}(3\sqrt{3}-1)$ (b) $3\sqrt{3}+1$

(c) $\frac{1}{\sqrt{3}}(2+7\sqrt{3})$ (d) none of these

30. If A, B, C are the angles which a directed line makes with the positive directions of the coordinate axes, then $\sin^2 A + \sin^2 B + \sin^2 C =$

(a) 1 (b) 2

(c) 3 (d) none of these

31. If sine of the angle between two vectors $-3\hat{i}+4\hat{j}+\hat{k}$ and $-2\hat{i}-\hat{j}-\hat{k}$ be $1+\frac{1}{2}x-\frac{1}{8}x^2 - ......$, to ∞ then $x =$

(a) $\frac{155}{156}$ (b) $\frac{1}{156}$

(c) $-\frac{1}{156}$ (d) none of these

32. The domain of the function $f(x) = \frac{\sqrt{9-x^2}}{\sin^{-1}(3-x)}$ is

(a) (2, 3) (b) [2, 3)

(c) (2, 3] (d) None of these

33. $\int_0^2 ([x]^2 - [x^2])\, dx$ is equal to

(a) $5-\sqrt{2}-\sqrt{3}$ (b) $\sqrt{2}+\sqrt{3}+4$

(c) $\sqrt{2}-\sqrt{3}+4$ (d) $\sqrt{2}+\sqrt{3}-4$

34. Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their point of intersection of given two circles and the point (1, 1) is

(a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$

(c) $x^2 + y^2 - 4y + 2 = 0$ (d) None of these

35. The coefficient of x^4 in $\left(\frac{x}{2}-\frac{3}{x^2}\right)^{10}$ is

(a) $\frac{405}{256}$ (b) $\frac{504}{259}$

(c) $\frac{450}{263}$ (d) none of these

36. If $z = \cos\theta + i\sin\theta$, then $\frac{z^{2n}-1}{z^{2n}+1} = ?$ (n is an integer)

(a) $i\cot n\theta$ (b) $i\tan n\theta$ (c) $\tan n\theta$ (d) $\cot n\theta$

37. Let $f(\theta) = 1 + \sin\theta$ and $a_n = \left\{f\left(\frac{\pi}{4}\right)\right\}^n + \left\{f\left(-\frac{\pi}{4}\right)\right\}^n$,

$b_n = \left\{f\left(\frac{\pi}{4}\right)\right\}^n - \left\{f\left(-\frac{\pi}{4}\right)^n\right\}$

then $a_{m+n} + K \cdot a_{m-n} = a_m a_n$ and $b_{m+n} - K \cdot b_{m-n} = a_m b_n$ both holds simultaneously if $K =$

(a) 1 (b) 2^{-n} (c) -2^n (d) -2^{-n}

38. The locus of chords of contact of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ touch another fixed ellipse is a/an

(a) circle (b) straight line

(c) ellipse (d) hyperbola

39. If $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}.....\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}.....\right)=\frac{\pi}{2}$, $0 < |x| < \sqrt{2}$, then $x =$

(a) $\frac{1}{2}$ (b) 1 (c) $-\frac{1}{2}$ (d) – 1

40. The sum to 20 terms of the series $1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + ...$ is

(a) 4410 (b) 4210 (c) 4120 (d) 4040

41. $\int \frac{xe^x}{(x+1)^2} dx =$

(a) $\frac{e^x}{x+1}$ (b) $\frac{e^x}{(x+1)^2}$

(c) $\frac{-e^x}{(x+1)^3}$ (d) none of these

42. If $\prod_{r=1}^{n} (1 + x + x^2 + ... + x^r)$ $= A_0 + A_1x + A_2x^2 + ...$ then

(a) $A_0 + A_2 + A_4 + ... = \frac{\underline{|n+1}}{2}$

(b) $A_1 + A_3 + A_5 + ... = \frac{\underline{|n+1}}{2}$

(c) $A_0 + A_1 + A_2 + A_3 + ... = \underline{|n+1}$

(d) all of these

43. Four students of class IV, 5 students of class V and 6 students of class VI sit in a row. The number of ways they can sit in a row so that the students belonging to same class are together is

(a) 4! 5! 6! (b) $\frac{15!}{4!5!6!}$

(c) $\frac{15!}{3!4!5!6!}$ (d) 3! 4! 5! 6!

44. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is

(a) 128 (b) 216 (c) 240 (d) 160

45. The entire graph of the equation $y = x^2 + kx - x + 9$ lies above the x-axis, then

(a) $k > 7$ (b) $-5 < k < 7$

(c) $k > -5$ (d) none of these

SOLUTIONS

1. (c) : The circle through P and Q is $S + \lambda S' = 0$

$(1 + \lambda)(x^2 + y^2) + (3 + 2\lambda)x + (7 + 2\lambda)y + 2p - 5 - \lambda p^2 = 0$

If it passes through (1, 1), then

$2(1 + \lambda) + 3 + 2\lambda + 7 + 2\lambda + 2p - 5 - \lambda p^2 = 0$

$$\Rightarrow 6\lambda - p^2\lambda + 7 + 2p = 0 \Rightarrow \lambda = \frac{7 + 2p}{p^2 - 6}$$

$\therefore\ p^2 - 6 \neq 0 \Rightarrow p \neq \sqrt{6}, -\sqrt{6}.$

2. (d) : The equation of the parabola is

$(x - 0)^2 + (y - 1)^2 = (x + 2)^2$

$\Rightarrow (y - 1)^2 = 4(x + 1).\ \therefore\ y - 1 = 2t, x + 1 = t^2$

$\therefore$ The point is $(t^2 - 1, 2t + 1)$.

3. (b) : Let $S_n = \frac{3}{1^2} + \frac{5}{1^2 + 2^2} + \frac{7}{1^2 + 2^2 + 3^2} + \ldots$

$$t_n = \frac{(2n+1)}{1^2 + 2^2 + 3^2 + \ldots + n^2} = \frac{6(2n+1)}{n(n+1)(2n+1)} = \frac{6}{n(n+1)}$$

$$= 6\left[\frac{(n+1) - n}{n(n+1)}\right] = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$$

Now, $S_n = t_1 + t_2 + \ldots + t_n$

$$= 6\left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right)\right]$$

$$= 6\left[1 - \frac{1}{n+1}\right] = 6\left\{\frac{(n+1) - 1}{(n+1)}\right\} = \frac{6n}{n+1}.$$

4. (c) : The pair of tangents, $T^2 = S_1S$.

$\Rightarrow (-x - 2(x - 1) + 4y)^2 = (1 + 4 + 4)(x^2 - 4x + 4y^2)$

$\Rightarrow (4y - 3x + 2)^2 - 9(x^2 - 4x + 4y^2) = 0$

$\Rightarrow 6xy + 5y^2 - 6x - 4y - 1 = 0$

Angle between the lines is $\tan^{-1}\left(\frac{2\sqrt{h^2 - ab}}{a+b}\right) = \tan^{-1}\frac{6}{5}$

$\therefore\ \tan\theta = \frac{6}{5}$.

5. (d) : Equation of the parabola is $y = x^2 - 4x + 3$

$\Rightarrow x^2 - 4x + 4 = y + 1 \Rightarrow (x - 2)^2 = (y + 1)$

$\therefore$ Vertex is $A(2, -1)$

Equation of the circle is $x^2 + (y - 3)^2 = 9$

$\therefore$ Centre of the circle is $C(0, 3)$

Now, the required distance, $AC = \sqrt{2^2 + (-4)^2} = 2\sqrt{5}$

6. (d) : We have $A = \frac{a+b}{2}$. Let $A_1, A_2, \ldots, A_n$, be n A.M.'s between a and b. Then $a, A_1, A_2, \ldots, A_n, b$ is an A.P. with common difference $d = \frac{b-a}{n+1}$.

Now, $S = (A_1 + A_2 + \ldots + A_n)$

$$= \frac{n}{2}(A_1 + A_n) = \frac{n}{2}(a+b) = nA \quad \therefore \quad \frac{S}{A} = n$$

7. (d) : $\cos^{-1}\frac{y}{b} = n\log\frac{x}{n}$

Differentiating w.r.t. x, we get

$$-\frac{1}{\sqrt{1 - \frac{y^2}{b^2}}}\frac{y_1}{b} = \frac{n}{x} \Rightarrow \frac{y_1^2}{b^2 - y^2} = \frac{n^2}{x^2}$$

$\Rightarrow x^2y_1^2 = n^2(b^2 - y^2)$

Differentiating again w.r.t. x, we get

$2x^2y_1y_2 + 2xy_1^2 = -2n^2yy_1$

Dividing both sides by $2y_1$, we get $x^2y_2 + xy_1 = -n^2y$

$\Rightarrow x^2y_2 = -xy_1 - n^2y$

8. (c) : If $n = 1$, then

$$\frac{n^2}{(n+1)^2} = \frac{1}{4}, \frac{n^3}{(n+1)^3} = \frac{1}{8}, \frac{n}{n+1} = \frac{1}{2}, \frac{1}{n+1} = \frac{1}{2}$$

If $n = 2$, then $\frac{n}{n+1} = \frac{2}{3}$ and $\frac{1}{n+1} = \frac{1}{3}$.

So, sum is $\frac{n}{n+1}$.

9. (c) : $\tan^{-1}\sqrt{x(x+1)}$ is real only if

$x(x + 1) \geq 0$ ...(i)

$\sin^{-1}\sqrt{x^2 + x + 1}$ is real only if $x^2 + x + 1 \leq 1$

or $x^2 + x \leq 0$ ...(ii)

(i), (ii) $\Rightarrow x^2 + x = 0 \Rightarrow x = 0, -1.$

10. (c) : Let $I = \int_0^1 f(k-1+x)dx$

Then $I = \int_{k-1}^{k} f(t)dt$, where $t = k - 1 + x$

$$\Rightarrow I = \int_{k-1}^{k} f(x)dx$$

$$\therefore \sum_{k=1}^{n} \int_{k-1}^{k} f(x)dx = \int_0^1 f(x)dx + \int_1^2 f(x)dx + ... + \int_{n-1}^{n} f(x)dx$$

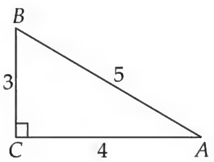
$$= \int_0^n f(x)dx$$

11. (a) : The chord is $T = S_1$

$\Rightarrow 12x - 45y = 36 - 225 \Rightarrow 4x - 15y + 63 = 0$

Distance of (0, 0) from it is $\dfrac{63}{\sqrt{4^2+15^2}} = \dfrac{63}{\sqrt{241}}$.

12. (c) : $\Delta = \dfrac{1}{2}\cdot 3\cdot 4 = 6$



$s = \dfrac{1}{2}(3+4+5) = 6$

$r_1 = \dfrac{\Delta}{s-a} = \dfrac{6}{6-3} = 2$

$r_2 = \dfrac{\Delta}{s-b} = \dfrac{6}{6-4} = 3,\ r_3 = \dfrac{\Delta}{s-c} = \dfrac{6}{6-5} = 6$

$\therefore\ r_1 + r_2 + r_3 = 2 + 3 + 6 = 11$

13. (c) : $\dfrac{x^{n+1}}{n+1}\log x - \dfrac{1}{(n+1)}\int x^{n+1}\cdot\dfrac{1}{x}dx$

(Applying by parts)

$$= \frac{x^{n+1}}{n+1}\log x - \frac{1}{(n+1)^2}\cdot x^{n+1} + C$$

14. (c) : $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{y}$

$$\Rightarrow \int \frac{y}{\sqrt{1-y^2}}dy = \int dx \Rightarrow -\sqrt{1-y^2} = x + C$$

$\Rightarrow 1 - y^2 = x^2 + 2xC + C^2$

$\Rightarrow x^2 + y^2 + 2xC + C^2 - 1 = 0$

The above equation represents a family of circles whose centre is $(-C, 0)$, radius $= \sqrt{C^2 - C^2 + 1} = 1$.

Thus, the given differential equation determines a family of circles with fixed radius 1 and variable centres along the x-axis.

15. (d) : Clearly p is false and q and r are true.

The option (d) is true because it is $\sim p \vee (q \wedge r)$ which means not p or (q and r) which means p is not correct and q and r are correct.

16. (b) : Let $I = \displaystyle\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}$

Put $t = 1 + \dfrac{1}{x^4}$

$$\therefore\ I = -\frac{1}{4}\int \frac{dt}{t^{\frac{3}{4}}} = -t^{\frac{1}{4}} + c = -\frac{(x^4+1)^{\frac{1}{4}}}{x} + c$$

17. (d) : The function for $x > 2$ can be redefined as

$$f(x) = \int_0^1 (1+(1-t))dt + \int_1^x (1+(t-1))dt$$

$$= 2(1) - \frac{1}{2} + \frac{x^2}{2} - \frac{1}{2} = \frac{x^2}{2} + 1$$

$$\text{R.H.L.} = \lim_{h\to 0}\left\{\frac{(2+h)^2}{2} + 1\right\} = 3$$

$$\text{L.H.L.} = \lim_{h\to 0}\{5(2-h) - 7\} = 3$$

$\because$ L.H.L = R.H.L $\Rightarrow f$ is continuous at $x = 2$

$$\text{Now, L.H.D.} = \lim_{h\to 0}\frac{f(2) - f(2-h)}{h}$$

$$= \lim_{h\to 0}\frac{3 - 5(2-h) + 7}{h} = 5$$

$$\text{R.H.D.} = \lim_{h\to 0}\frac{f(2+h) - f(2)}{h} = \lim_{h\to 0}\frac{\frac{(2+h)^2}{2} + 1 - 3}{h}$$

$$= \lim_{h\to 0}\left\{\frac{h^2 + 4h + 4 + 2 - 6}{2h}\right\} = \lim_{h\to 0}\left[\frac{h}{2} + 2\right] = 2$$

$\because$ L.H.D. $\neq$ R.H.D.

$\therefore$ f is non-differentiable at $x = 2$.

18. (b) : Since, we have fifty real numbers, of which five numbers are to be picked.

$\therefore$ Total number of outcomes = ${}^{50}C_5$

Also numbers are picked in such way that x_{20} is the middle number *i.e.*, we select 2 numbers from first nineteen numbers and 2 numbers from last thirty numbers.

$\therefore$ Number of favourable outcomes = ${}^{19}C_2 \times {}^{30}C_2$

$\therefore$ Required probability = $\dfrac{{}^{19}C_2 \times {}^{30}C_2}{{}^{50}C_5}$

19. (b) : $f(x) = x^3 \operatorname{sgn} x = \begin{cases} x^3\dfrac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

So, $f(x) = x^2x = x^3$ for $x > 0$

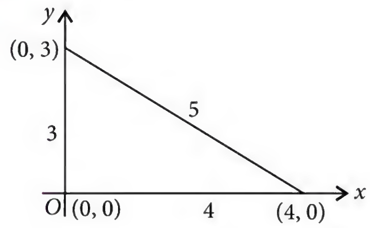
and $f(x) = x^2(-x) = -x^3$ for $x < 0$ and $f(0) = 0$.

$$Lf'(0) = \lim_{h\to 0^-}\frac{f(h)-f(0)}{h} = \lim_{h\to 0^-}\frac{-h^3-0}{h} = 0$$

and $$Rf'(0) = \lim_{h\to 0^+}\frac{f(h)-f(0)}{h} = \lim_{h\to 0^+}\frac{h^3-0}{h} = 0$$

So, f is derivable at 0.

20. (b) :



The incentre is given by $\left(\frac{\sum ax_1}{\sum a}, \frac{\sum ay_1}{\sum a}\right)$

$$= \left(\frac{0+0+3\times 4}{12}, \frac{0+0+4\times 3}{12}\right) = (1, 1)$$

21. (d) : By cosine rule

$$\frac{2(b^2+c^2-a^2)+(c^2+a^2-b^2)+2(a^2+b^2-c^2)}{2abc} = \frac{a^2+b^2}{abc}$$

$\therefore\ 3b^2 + c^2 + a^2 = 2a^2 + 2b^2 \Rightarrow a^2 = b^2 + c^2$

$\Rightarrow A = 90°$

22. (d) : Let n_1 and n_2 be the number of observations in two groups having means $\bar{X}_1$ and $\bar{X}_2$ respectively. Then

$$\bar{X} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2}$$

Now, $$\bar{X} - \bar{X}_1 = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1+n_2} - \bar{X}_1 = \frac{n_2(\bar{X}_2 - \bar{X}_1)}{n_1+n_2} > 0 \quad [\because\ \bar{X}_2 > \bar{X}_1]$$

$\Rightarrow\ \bar{X} > \bar{X}_1$ ...(i)

And, $\bar{X} - \bar{X}_2 = \frac{n_1(\bar{X}_1 - \bar{X}_2)}{n_1+n_2} < 0 \quad [\because\ \bar{X}_2 > \bar{X}_1]$

$\Rightarrow\ \bar{X} < \bar{X}_2$ ...(ii)

From (i) and (ii), $\bar{X}_1 < \bar{X} < \bar{X}_2$.

23. (b) : $y = \sqrt{\sin x + y} \Rightarrow y^2 = \sin x + y$

$$\Rightarrow 2y\cdot\frac{dy}{dx} = \cos x + \frac{dy}{dx} \quad \therefore\ \frac{dy}{dx} = \frac{\cos x}{2y-1}$$

24. (d) : We have $x^2 - 3x + k = 0$

Given that roots are real and distinct.

$\therefore\ (-3)^2 - 4\times 1\times k > 0$ or $k < \frac{9}{4}$

Sum of roots $= -\frac{(-3)}{1} = 3$

Since each root lies in (0, 1), the sum of roots cannot be 3.

$\therefore$ No value of k can give desired roots.

25. (d) : We have

$$S = \sum_{n=2}^{\infty}\frac{{}^nC_2}{(n+1)!} = \sum_{n=2}^{\infty}\frac{n!}{(n-2)!(n+1)!\ 2!}$$

$$= \frac{1}{2}\sum_{n=2}^{\infty}\frac{n(n-1)}{(n+1)!} = \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{n^2-1-n-1+2}{(n+1)!}\right)$$

$$= \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{n-1}{n!} - \frac{1}{n!} + \frac{2}{(n+1)!}\right)$$

$$= \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{(n-1)!} - \frac{2}{n!} + \frac{2}{(n+1)!}\right)$$

$$= \frac{1}{2}\left[(e-1) - 2(e-2) + 2\left(e - \frac{5}{2}\right)\right] = \frac{e}{2} - 1$$

26. (c) : $\because\ -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}, -\frac{\pi}{2} \le \sin^{-1}y \le \frac{\pi}{2}$

and $-\frac{\pi}{2} \le \sin^{-1}z \le \frac{\pi}{2}$

Given that, $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

which is possible only when

$\sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

or $x = y = z = 1$.

Put $p = q = 1$ then, $f(2) = f(1)f(1) = 2\cdot 2 = 4$

and put $p = 1, q = 2$, then, $f(3) = f(1)f(2) = 2\cdot 2^2 = 8$

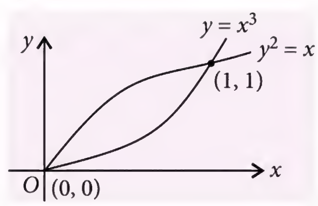
$$\therefore\ x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$$

$$= 1+1+1 - \frac{3}{1+1+1} = 3 - 1 = 2$$

27. (a) : Required area

$$= \int_0^1 (y_1 - y_2)\,dx$$

$$= \int_0^1 (\sqrt{x} - x^3)dx = \frac{5}{12}$$



28. (c) : Let $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sin x + \cos x} dx$

$= \int_0^{\pi/2} (\sin x + \cos x) dx = 1 + 1 = 2.$

29. (a) : Putting $y = x\sqrt{3}$ in the equation of curve, we get $x^3(3\sqrt{3}+1) + x^2(3\sqrt{3}+14) + (4+5\sqrt{3})x - 1 = 0$

If the roots of this equation are x_1, x_2 and x_3

$\Rightarrow \quad x_1x_2x_3 = \frac{1}{3\sqrt{3}+1} = \frac{3\sqrt{3}-1}{26}$

Also, for a point A, coordinates are $(x_1, x_1\sqrt{3})$

$\therefore \quad OA = \sqrt{x_1^2 + 3x_1^2} = 2x_1$

$\Rightarrow OA \cdot OB \cdot OC = (2x_1)(2x_2)(2x_3) = 8x_1x_2x_3$

$= \frac{4}{13}(3\sqrt{3}-1)$

30. (b) : $\Sigma\cos^2 A = 1 \;\Rightarrow\; \Sigma(1-\sin^2 A) = 1$

$\Rightarrow \Sigma\sin^2 A = 2$

31. (c) : $\cos\theta = \frac{(-3\hat{i}+4\hat{j}+\hat{k}).(-2\hat{i}-\hat{j}-\hat{k})}{\sqrt{9+16+1}\sqrt{4+1+1}}$

$= \frac{6-4-1}{\sqrt{156}} = \frac{1}{\sqrt{156}}$

$\therefore \quad \sin\theta = \sqrt{1-\frac{1}{156}} = \left(1-\frac{1}{156}\right)^{1/2}$

$= 1 + \frac{1}{2}\left(-\frac{1}{156}\right) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(-\frac{1}{156}\right)^2 + \text{...... to } \infty$

$= 1 + \frac{1}{2}\left(-\frac{1}{156}\right) - \frac{1}{8}\left(-\frac{1}{156}\right)^2 + \text{...... to } \infty$

On comparison, we get $x = -\frac{1}{156}$

32. (b) : $\sqrt{9-x^2}$ is defined for $9 - x^2 \geq 0$

$\Rightarrow \quad (3-x)(3+x) \geq 0 \;\Rightarrow\; -3 \leq x \leq 3$ ...(i)

$\sin^{-1}(3-x)$ is defined for $-1 \leq 3 - x \leq 1$

$\Rightarrow \quad -4 \leq -x \leq -2 \;\Rightarrow\; 2 \leq x \leq 4$ ...(ii)

Also, $\sin^{-1}(3-x) \neq 0 \Rightarrow 3 - x \neq 0$ or $x \neq 3$ ...(iii)

From (i), (ii) and (iii), we get

the domain of $f = ([-3, 3] \cap [2, 4]) - \{3\} = [2, 3)$.

33. (d) : $\int_0^2 ([x]^2 - [x^2])\, dx = \int_0^2 [x]^2\, dx - \int_0^2 [x^2]\, dx$

$= \left(\int_0^1 [x]^2 dx + \int_1^2 [x]^2\, dx\right)$

$-\left(\int_0^1 [x^2] dx + \int_1^{\sqrt{2}} [x^2]\, dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]\, dx + \int_{\sqrt{3}}^2 [x^2]\, dx\right)$

$= 0 + \int_1^2 1^2\, dx - \left(0 + \int_1^{\sqrt{2}} 1\, dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\, dx + \int_{\sqrt{3}}^2 3\, dx\right)$

$= (2-1) - \{(\sqrt{2}-1) + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3})\}$

$= 1 - \{\sqrt{2} - 1 + 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}\} = \sqrt{2} + \sqrt{3} - 4$

34. (b) : Equation of circle passing through the point of intersection of given circles, is

$x^2 + y^2 - 6x + 8 + \lambda(x^2 + y^2 - 6) = 0$ ...(i)

This circle passes through (1, 1) then

$1 + 1 - 6 + 8 + \lambda(1 + 1 - 6) = 0 \Rightarrow \lambda = 1$

On putting the value of λ in equation (i), we get

$2x^2 + 2y^2 - 6x + 2 = 0 \Rightarrow x^2 + y^2 - 3x + 1 = 0$

which is the required equation of circle.

35. (a) : General term of given expansion is given by

$\binom{10}{r}\left(\frac{x}{2}\right)^{10-r}\left(-\frac{3}{x^2}\right)^r = \binom{10}{r}2^{r-10}(-3)^r \cdot x^{10-3r}$

For coeff. of x^4, $10 - 3r = 4 \Rightarrow r = 2$

$\therefore$ The required coefficient is $\binom{10}{2}2^{-8} \cdot 9 = \frac{405}{256}$.

36. (b) : We have, $\frac{z^{2n}-1}{z^{2n}+1} = \frac{(\cos\theta + i\sin\theta)^{2n} - 1}{(\cos\theta + i\sin\theta)^{2n} + 1}$

$= \frac{\cos 2n\theta + i\sin 2n\theta - 1}{\cos 2n\theta + i\sin 2n\theta + 1}$ (Using De Moivre's theorem)

$= \frac{(1 - 2\sin^2 n\theta) + 2i\sin n\theta\cos n\theta - 1}{(2\cos^2 n\theta - 1) + 2i\sin n\theta\cos n\theta + 1}$

$= \frac{i\sin n\theta\cos n\theta + i^2\sin^2 n\theta}{\cos^2 n\theta + i\sin n\theta\cos n\theta} \quad (\because \; i^2 = -1)$

$= \frac{i\sin n\theta(\cos n\theta + i\sin n\theta)}{\cos n\theta(\cos n\theta + i\sin n\theta)} = i\tan n\theta$

37. (b) : $\because \quad a_n = \left\{f\left(\frac{\pi}{4}\right)\right\}^n + \left\{f\left(-\frac{\pi}{4}\right)\right\}^n$

$\therefore \quad a_n = \left(1+\frac{1}{\sqrt{2}}\right)^n + \left(1-\frac{1}{\sqrt{2}}\right)^n = x^n + y^n$ (say)

$\therefore \quad xy = 1 - \frac{1}{2} = \frac{1}{2}$

Similarly, $b_n = x^n - y^n$

Now, $a_m a_n - a_{m+n} = (x^m + y^m)(x^n + y^n) - (x^{m+n} + y^{m+n})$

$= x^n y^m + x^m y^n = (xy)^n(x^{m-n} + y^{m-n})$

$= \left(\frac{1}{2}\right)^n . a_{m-n} = 2^{-n} . a_{m-n}$

On comparing, we get $K = 2^{-n}$

Similarly, 2nd relation $b_{m+n} - K \cdot b_{m-n} = a_m \cdot b_n$ holds for $K = 2^{-n}$ also.

38. (c) : Locus of the point of intersection of perpendicular tangents to the given ellipse is
$x^2 + y^2 = a^2 + b^2$
Any point on this circle can be taken as
$$P \equiv \left(\sqrt{a^2+b^2}\cos\theta,\ \sqrt{a^2+b^2}\sin\theta\right)$$
The equation of the chord of contact of tangents from P is
$$\frac{x}{a^2}\sqrt{a^2+b^2}\cos\theta + \frac{y}{b^2}\sqrt{a^2+b^2}\sin\theta = 1$$
Let this line be a tangent to the fixed ellipse
$$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1 \Rightarrow \frac{x}{A}\cos\theta + \frac{y}{B}\sin\theta = 1$$
where $A = \dfrac{a^2}{\sqrt{a^2+b^2}},\ B = \dfrac{b^2}{\sqrt{a^2+b^2}}$
$$\frac{x^2}{a^4}+\frac{y^2}{b^4} = \frac{1}{(a^2+b^2)}$$, which is an ellipse.

39. (b) : Summing the infinite G.P.'s,
$$x - \frac{x^2}{2} + \frac{x^3}{4} \ldots\ldots = \frac{x}{1+\frac{x}{2}} = \frac{2x}{2+x}$$
$$x^2 - \frac{x^4}{2} + \frac{x^6}{4} \ldots\ldots = \frac{x^2}{1+\frac{x^2}{2}} = \frac{2x^2}{2+x^2}$$
As, $\sin^{-1}\alpha + \cos^{-1}\alpha = \dfrac{\pi}{2}$
$$\Rightarrow \frac{2x}{2+x} = \frac{2x^2}{2+x^2} \Rightarrow 2 + x^2 = 2x + x^2$$
$\therefore\ 2 = 2x \Rightarrow x = 1$ since $x \neq 0$.

40. (a) : Let $S = (1^2 + 2^2 + \ldots + 20^2) + (2^2 + 4^2 + \ldots + 20^2)$
$$= \frac{20 \times 21 \times 41}{6} + 4(1^2 + 2^2 + \ldots + 10^2)$$
$$= 2870 + 1540 = 4410.$$

41. (a) : $I = \displaystyle\int \frac{e^x(x+1-1)}{(x+1)^2}dx$
$$= \int e^x\left[\frac{1}{x+1} - \frac{1}{(1+x)^2}\right]dx = e^x\left(\frac{1}{x+1}\right)$$
$[\because$ It is of the form $\int e^x(f(x)+f'(x))dx]$

42. (d) : Here, $(1+x)(1+x+x^2)(1+x+x^2+x^3)$ ...... $(1+x+x^2+\ldots\ldots+x^n)$
$= A_0 + A_1x + A_2x^2 + \ldots\ldots$ ...(i)

In L.H.S., highest power of $x = 1 + 2 + 3 + \ldots. + n$
$$= \frac{n(n+1)}{2}$$
$\therefore$ R.H.S. of (i) $= A_0 + A_1x + A_2x^2 + \ldots + A_{\frac{n(n+1)}{2}} \cdot x^{\frac{n(n+1)}{2}}$
$\therefore$ On putting $x = 1$ in (i), we get
$2\cdot3\cdot4 \ldots\ldots (n+1) = A_0 + A_1 + A_2 + \ldots + A_{\frac{n(n+1)}{2}}$
$\therefore\ A_0 + A_1 + A_2 + A_3 + \ldots.. = \underline{|n+1}$ ...(ii)
Again, on putting $x = -1$ in (i), we get
$0 = A_0 - A_1 + A_2 - A_3 + \ldots$
$\therefore\ A_0 + A_2 + A_4 + A_6 + \ldots. = A_1 + A_3 + A_5 + \ldots.$ ...(iii)
On comparing (ii) & (iii), we get
$$A_0 + A_2 + A_4 + \ldots\ldots = A_1 + A_3 + A_5 + \ldots. = \frac{\underline{|n+1}}{2}$$

43. (d) : Class IV students can sit in 4! ways. Class V students in 5! ways and Class VI students in 6! ways. The classes can be permuted in 3! ways.
$\therefore$ The desired number is 3! 4! 5! 6!.

44. (d) : $n(C) = 224, n(H) = 240, n(B) = 336, n(H \cap B) = 64$
$n(B \cap C) = 80, n(H \cap C) = 40, n(C \cap H \cap B) = 24$
$\therefore\ n(C^c \cap H^c \cap B^c) = n[(C \cup H \cup B)^c]$
$= 800 - [224 + 240 + 336 - 64 - 80 - 40 + 24]$
$= 800 - 640 = 160$

45 (b) : $x^2 + (k-1)x + 9 > 0\ \forall\ x$
$\Delta < 0 \Rightarrow (k-1)^2 - 36 < 0 \Rightarrow (k-1+6)(k-1-6) < 0$
$\Rightarrow (k+5)(k-7) < 0 \Rightarrow k \in (-5, 7).$

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main cum Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main cum Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. The period of $\sin^4 x + \cos^4 x$ is

(a) 2π (b) π (c) $\pi/2$ (d) $\pi/4$

2. The range of the function $f(x) = 3|\sin x| - 2|\cos x|$ is

(a) $[-2, \sqrt{13}]$ (b) $[-2, 3]$

(c) $[3, \sqrt{13}]$ (d) $[-3, \sqrt{13}]$

3. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$. Then the value of $6A + 5B + 4C + 3D + 2E$ is

(a) -17 (b) -14 (c) -11 (d) -8

4. The coefficient of $a^4 b^5 c^3$ in the expansion of $(a + b + c)^{10}$ is

(a) $\frac{(10)!}{4!5!3!}$ (b) $(10)!$ (c) 0 (d) $4!5!3!$

5. Minimum distance between the curves $y^2 = 4x$ and $x^2 + y^2 - 12x + 31 = 0$ is equal to

(a) $\sqrt{20}$ (b) $\sqrt{26} - \sqrt{5}$

(c) $\sqrt{20} - \sqrt{5}$ (d) $\sqrt{5} + \sqrt{20}$

6. $\int \frac{3 + 2\cos x}{(2 + 3\cos x)^2} dx =$

(a) $\frac{\sin x}{(2 + 3\cos x)^2} + c$ (b) $\frac{\cos x}{2 + 3\cos x} + c$

(c) $\frac{\sin x}{2 + 3\cos x} + c$ (d) $\frac{-\sin x}{2 + 3\cos x} + c$

7. The area of the region bounded by the curves $y = ex \log x$ and $y = \frac{\log x}{ex}$ is

(a) $\frac{e^2 - 5}{4e}$ (b) $e + \frac{5}{4}$

(c) $\frac{e}{4} - 5$ (d) $\frac{e}{4} - \frac{1}{4e}$

8. $\lim_{n \to \infty} \left(\cos\frac{x}{2} \cos\frac{x}{4} \cos\frac{x}{8} \ldots\ldots \cos\frac{x}{2^n} \right) =$

(a) $\frac{x}{\sin x}$ (b) $\frac{\sin x}{x}$ (c) 0 (d) 1

9. $\int \frac{\cos x}{1 - \sin x \cos x} dx$

$= \tan^{-1}(\sin x - \cos x) + \frac{k}{\sqrt{3}} \ln \left| \frac{\sin x + \cos x - \sqrt{3}}{\sin x + \cos x + \sqrt{3}} \right| + c$

where k is

(a) $-1/2$ (b) $1/2$ (c) -1 (d) 1

10. The pair of lines $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ is rotated about the origin by $\frac{\pi}{6}$ in the anti-clockwise sense. The equation of the pair in the new position is

(a) $x^2 - \sqrt{3}\,xy = 0$ (b) $xy - \sqrt{3}y^2 = 0$

(c) $\sqrt{3}x^2 - xy = 0$ (d) $x^2 = y^2$

By : **Prof. Shyam Bhushan,** Director, Narayana IIT Academy, Jamshedpur. Mob. : 09334870021

SOLUTIONS

1. **(c) :** $\sin^4 x+\cos^4 x=\frac{3}{4}+\frac{1}{4}\cos 4x$

Period $=\frac{2\pi}{4}=\frac{\pi}{2}$

2. **(b) :** $g(\alpha)=3\alpha-2\sqrt{1-\alpha^2}$ where $\alpha\in[0,1]$

[where $\alpha = |\sin x|$]

$g'(\alpha)=3+\frac{2\alpha}{\sqrt{1-\alpha^2}}>0\ \forall\alpha\in[0,1]$

Also, $g(0) = -2$, $g(1) = 3$

$\therefore$ Range = [–2, 3]

3. **(c) :** Let $\Delta(x) = \begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}$

$\Rightarrow\ \Delta(1) = 0,\ \Delta'(1) = -11$

4. **(c) :** $\because$ 4 + 5 + 3 = 12

5. **(c) :** Equation of normal is $y + tx = 2t + t^3$

It passes through centre of circle $C(6,0)$

$\Rightarrow\ t = 0,\ t = \pm 2$

Points $(4, \pm 4)$

$CA=CB=\sqrt{20}$

$\Rightarrow$ Minimum distance $=CA-\text{radius}=\sqrt{20}-\sqrt{5}$

6. **(c) :** Given integral is $\int\frac{d}{dx}\left(\frac{\sin x}{2+3\cos x}\right)$

7. **(d) :** Area $=\left|\int_{1/e}^{1}\left(ex\log x-\frac{\log x}{ex}\right)dx\right|$

8. **(b) :** $\lim_{n\to\infty}\left(\cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}........\cos\frac{x}{2^n}\right)$

$$=\lim_{n\to\infty}\frac{1}{2\sin\frac{x}{2^n}}\left\{\cos\frac{x}{2}\cos\frac{x}{4}....2\sin\frac{x}{2^n}\cos\frac{x}{2^n}\right\}$$

$$=\lim_{n\to\infty}\frac{1}{2^2\sin\frac{x}{2^n}}\left\{\cos\frac{x}{2}\cos\frac{x}{4}....\left(2\cos\frac{x}{2^{n-1}}\sin\frac{x}{2^{n-1}}\right)\right\}$$

$$=\lim_{n\to\infty}\frac{1}{2^n\sin\frac{x}{2^n}}\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)=\lim_{n\to\infty}\frac{\sin x}{2^n\sin\frac{x}{2^n}}=\frac{\sin x}{x}$$

9. **(a) :** $I=\frac{1}{2}\int\frac{\cos x+\sin x+\cos x-\sin x}{1-\sin x\cos x}dx$

$$I=\frac{1}{2}\int\frac{\cos x+\sin x}{1-\sin x\cos x}dx+\frac{1}{2}\int\frac{\cos x-\sin x}{1-\sin x\cos x}dx$$

$$=\frac{1}{2I_1}+\frac{1}{2I_2}$$

where $I_1=\int\frac{\sin x+\cos x}{1-\sin x\cos x}dx=\int\frac{2dt}{1+t^2}=2\tan^{-1}t$

$= 2\tan^{-1}(\sin x - \cos x)$, [Put $t = \sin x - \cos x$]

$I_2=\int\frac{\cos x-\sin x}{1-\sin x\cos x}$, put $t=\sin x+\cos x$

$$\int\frac{2dt}{3-t^2}=-\int\frac{2dt}{t^2-(\sqrt{3})^2}=-2\frac{1}{2\sqrt{3}}\log\left|\frac{\sin x+\cos x-\sqrt{3}}{\sin x+\cos x+\sqrt{3}}\right|$$

$\therefore\ k=\frac{-1}{2}$

10. **(c) :** Given pair of lines are

$(\sqrt{3}x-y)(x-\sqrt{3}y)=0$ or $y = \tan 60°x$, $y = \tan 30°x$

After rotation, the separate equations are

$y = \tan 90°x$, $y = \tan 60°x$

$x=0,\ y=\sqrt{3}x$

$\therefore$ Combined equation is $x(\sqrt{3}x-y)=0$

or $\sqrt{3}x^2-xy=0$

GEAR UP FOR JEE MAIN

EXAM ON
8th April (Offline)
15th & 16th April (Online)
2018

1. The equation of the circle passing through the focii of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at (0, 3) is

(a) $x^2 + y^2 - 6y + 7 = 0$ (b) $x^2 + y^2 - 6y - 5 = 0$
(c) $x^2 + y^2 - 6y + 5 = 0$ (d) $x^2 + y^2 - 6y - 7 = 0$

2. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are unit vectors equally inclined to each other at an angle α ($\neq 0$) then the angle between $\vec{a}\times\vec{b}$ and the plane containing $\vec{b}$ and $\vec{c}$ is

(a) $\sin^{-1}\left(\tan\frac{\alpha}{2}\cdot|\cot\alpha|\right)$ (b) $\cos^{-1}\left(\tan\frac{\alpha}{2}\cdot|\cot\alpha|\right)$

(c) $\cos^{-1}\left(\cot\frac{\alpha}{2}\cdot|\tan\alpha|\right)$ (d) $\sin^{-1}\left(\cot\frac{\alpha}{2}\cdot|\tan\alpha|\right)$

3. If a is the A.M of two numbers b and c and G_1, G_2 are two geometric means between b and c, then the value of $G_1^3+G_2^3=$

(a) 3 abc (b) 2 abc
(c) 4 abc (d) none of these

4. If O is the circum-centre of the triangle ABC and R_1, R_2, R_3 and R are the radii of the circum-circles of the triangle OBC, OCA, OAB and ABC respectively, then $\frac{a}{R_1}+\frac{b}{R_2}+\frac{c}{R_3}$ is equal to

(a) $\frac{abc}{R}$ (b) $\frac{abc}{R^3}$ (c) $\frac{abc}{R^4}$ (d) $\frac{abc}{R^2}$

5. The value of 'a' for which the system of equations $(a+1)^3x+(a+2)^3y=(a+3)^3$, $(a+1)x+(a+2)y=a+3$ and $x + y = 1$ is consistent, is

(a) -2 (b) -1 (c) 1 (d) 2

6. The area between the curve $y^2(2 - x) = x^3$ and its asymptote is

(a) $\pi/2$ (b) π (c) 2π (d) 3π

7. Let $f(x) = \cos^{-1}(\cos x)$ and $g(x) = \sin^{-1}[x+1]+\cos^{-1}[x]$ ($[x]$ = the greatest integer function), then the equation $f(x) + g(x) = 3$, has

(a) exactly one solution (b) exactly three solutions
(c) no solution (d) infinitely many solutions

8. Let AD be the angular bisector of the angle A of ΔABC, then $\overrightarrow{AD}=\alpha\overrightarrow{AB}+\beta\overrightarrow{AC}$, where

(a) $\alpha=\frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}+\overrightarrow{AC}|}$, $\beta=\frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}+\overrightarrow{AC}|}$

(b) $\alpha=\frac{|\overrightarrow{AB}|+|\overrightarrow{AC}|}{|\overrightarrow{AB}|}$, $\beta=\frac{|\overrightarrow{AB}|+|\overrightarrow{AC}|}{|\overrightarrow{AC}|}$

(c) $\alpha=\frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$, $\beta=\frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$

(d) $\alpha=\left|\frac{\overrightarrow{AB}}{\overrightarrow{AC}}\right|$, $\beta=\left|\frac{\overrightarrow{AC}}{\overrightarrow{AB}}\right|$

9. If $\int f(x)dx=\psi(x)$ then $\int x^5 f(x^3)\,dx$ is equal to

(a) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(b) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$

(c) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

(d) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$

10. Minimum distance between $y^2-4x-8y+40=0$ and $x^2-8x-4y+40=0$ is

(a) 0 (b) $\sqrt{3}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$

11. If $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit ortho-normal vectors and $\vec{a}$ is a vector, such that $\vec{a}\times\vec{r}=\hat{j}$, then $\vec{a}\cdot\vec{r}$ is equal to, for any vector $\vec{r}$

(a) 0 (b) 1
(c) -1 (d) arbitrary scalar

12. If $I_{m,n}=\int_0^1 t^m(1+t)^n\,dt$, then $I_{m,n}=$

(a) $\frac{2^n}{m+1}+\frac{n}{m+1}I_{m+1,n-1}$

(b) $\frac{n}{m+1}I_{m+1,n-1}$ (c) $\frac{2^n}{m+1}-\frac{n}{m+1}I_{m+1,n-1}$

(d) none of these

13. If $u = e^{\sin^{-1}x}$, $v = \log x$, then $\frac{du}{dv} =$

(a) xu (b) $\frac{u}{\sqrt{1-x^2}}$

(c) $\frac{xu}{\sqrt{1-x^2}}$ (d) none of these

14. The line $2px + y\sqrt{1-p^2} = 1$ $(|p|<1)$ for different values of p touches

(a) an ellipse of eccentricity $\frac{2}{\sqrt{3}}$

(b) an ellipse of eccentricity $\frac{\sqrt{3}}{2}$

(c) hyperbola of eccentricity 2

(d) none of these

15. $\lim_{x\to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x}-1} =$

(a) 0 (b) 1/2 (c) 2 (d) 3

16. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$ is equal to

(a) 1/2 (b) 1 (c) 2 (d) –1/4

17. If p, q, r are in A.P., then the determinant

$$\begin{vmatrix} a^2+2^{2n+1}+2p & b^2+2^{n+2}+3q & c^2+p \\ 2^n+p & 2^{n+1}+q & 2q \\ a^2+2^n+p & b^2+2^{n+1}+2q & c^2-r \end{vmatrix} =$$

(a) 1 (b) 0

(c) $a^2b^2c^2 - 2^n$ (d) $(a^2+b^2+c^2) - 2^nq$

18. Let f be a non negative function defined on the interval [0, 1]. If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$ and $f(0) = 0$, then

(a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(b) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

(c) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

(d) $f\left(\frac{1}{2}\right) > 0$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

19. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(a) 75 (b) 150 (c) 210 (d) 243

20. Equation of the line of shortest distance between the lines, $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ is

(a) $3(x-21) = 3y+92 = 3z-32$

(b) $\frac{x-(62/3)}{1/3} = \frac{y-31}{1/3} = \frac{z+(31/3)}{1/3}$

(c) $\frac{x-21}{1/3} = \frac{y-(92/3)}{1/3} = \frac{z+(32/3)}{1/3}$

(d) $\frac{x-2}{1/3} = \frac{y+3}{1/3} = \frac{z-1}{1/3}$

21. Let $f(x) = \begin{cases} x^2\left|\cos\frac{\pi}{x}\right|, & x \ne 0 \\ 0, & x = 0 \end{cases}, x \in R$, then f is

(a) differentiable both at $x = 0$ and at $x = 2$

(b) differentiable at $x = 0$ but not differentiable at $x = 2$

(c) not differentiable at $x = 0$ but differentiable at $x = 2$

(d) differentiable at neither at $x = 0$ nor at $x = 2$

22. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is

(a) $\sqrt{45}$ (b) $\sqrt{18}$ (c) $\sqrt{72}$ (d) $\sqrt{33}$

23. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(a) one-one and onto (b) onto but not one-one

(c) one-one but not onto

(d) neither one-one nor onto

24. The point P is the intersection of the straight line joining the points $Q(2, 3, 5)$ and $R(1, -1, 4)$ with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drawn from the point $T(2, 1, 4)$ to QR, then the length of the line segment PS is

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $2\sqrt{2}$

25. The domain of the function

$$f(x) = \sqrt{\log_{\sin x + \cos x}(|\cos x| + \cos x)},\ 0 \le x \le \pi \text{ is}$$

(a) $(0, \pi)$ (b) $\left(0, \frac{\pi}{2}\right)$

(c) $\left(0, \frac{\pi}{3}\right]$ (d) none of these

26. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(a) $\frac{91}{216}$ (b) $\frac{108}{216}$ (c) $\frac{125}{216}$ (d) $\frac{127}{216}$

27. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(a) 2^{10} (b) 2^{11} (c) 2^{12} (d) 2^{13}

28. If z is a complex number of unit modulus and argument θ, then $\arg\left(\dfrac{1+z}{1+\bar{z}}\right)$ equals

(a) $\dfrac{\pi}{2} - \theta$ (b) θ (c) $\pi - \theta$ (d) $-\theta$

29. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $(\sqrt[3]{1+a} - 1)x^2 + (\sqrt{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$ where $a > -1$. Then $\lim\limits_{a \to 0^+} \alpha(a)$ and $\lim\limits_{a \to 0^+} \beta(a)$ are

(a) $-\dfrac{5}{2}$ and 1 (b) $-\dfrac{1}{2}$ and -1

(c) $-\dfrac{7}{2}$ and 2 (d) $-\dfrac{9}{2}$ and 3

30. The points on the axis of the parabola $3y^2 + 4y - 6x + 8 = 0$ from where 3 distinct normals can be drawn is given by

(a) $\left(a, \dfrac{4}{3}\right)$; $a > \dfrac{19}{9}$ (b) $\left(a, -\dfrac{2}{3}\right)$; $a > \dfrac{19}{9}$

(c) $\left(a, -\dfrac{2}{3}\right)$; $a > \dfrac{16}{9}$ (d) $\left(a, -\dfrac{2}{3}\right)$; $a > \dfrac{7}{9}$

SOLUTIONS

1. (d): Foci are given by $(\pm ae, 0)$

As $a^2e^2 = a^2 - b^2 = 7$ we have equation of circle as

$(x - 0)^2 + (y - 3)^2 = (\sqrt{7} - 0)^2 + (0 - 3)^2$

$\therefore \quad x^2 + y^2 - 6y - 7 = 0$

2. (a): Let θ be the required angle, then

$$\sin\theta = \cos(90° - \theta) = \left|\frac{(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})}{|\vec{a} \times \vec{b}| \cdot |\vec{b} \times \vec{c}|}\right|$$

$$= \left|\frac{[(\vec{a} \times \vec{b}) \times \vec{b}] \cdot \vec{c}}{\sin^2\alpha}\right| = \left|\frac{[(\vec{a} \cdot \vec{b})\vec{b} - (\vec{b} \cdot \vec{b})\vec{a}] \cdot \vec{c}}{\sin^2\alpha}\right|$$

$$= \left|\frac{\cos\alpha \vec{b} \cdot \vec{c} - \vec{a} \cdot \vec{c}}{\sin^2\alpha}\right| = \left|\frac{\cos^2\alpha - \cos\alpha}{\sin^2\alpha}\right|$$

$$= \left|-\frac{\cos\alpha(1 - \cos\alpha)}{\sin\alpha \sin\alpha}\right| = \left|-\frac{\cos\alpha \cdot 2\sin^2(\alpha/2)}{\sin\alpha \cdot 2\sin(\alpha/2)\ \cos(\alpha/2)}\right|$$

$$= \left|-\cot\alpha \tan\frac{\alpha}{2}\right| = \left|\cot\alpha \tan\frac{\alpha}{2}\right|$$

$$\therefore \quad \theta = \sin^{-1}\left(\tan\frac{\alpha}{2} |\cot\alpha|\right)$$

3. (b)

4. (b): $R_1 = \dfrac{a}{2\sin 2A} \quad \Rightarrow \quad \dfrac{a}{R_1} = 2\sin 2A$

Similarly, $\dfrac{b}{R_2} = 2\sin 2B$, $\dfrac{c}{R_3} = 2\sin 2C$

$$\therefore \quad \frac{a}{R_1} + \frac{b}{R_2} + \frac{c}{R_3} = 2(\sin 2A + \sin 2B + \sin 2C)$$

$$= 2(4\sin A \sin B \sin C) = \frac{abc}{R^3}$$

5. (a): The system of equations is consistent, if the determinant of coefficients is zero.

$$\Rightarrow \begin{vmatrix} (a+1)^3 & (a+2)^3 & (a+3)^3 \\ a+1 & a+2 & a+3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_2$, we get

$$\begin{vmatrix} (a+1)^3 & 3a^2 + 9a + 7 & 3a^2 + 15a + 19 \\ a+1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

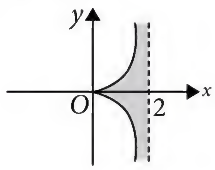
$\Rightarrow \quad (3a^2 + 9a + 7) - (3a^2 + 15a + 19) = 0$

$\Rightarrow \quad -6a - 12 = 0$ or $a = -2$

6. (d): The curve is symmetric about x-axis with asymptote $x = 2$.

$\therefore \quad$ Required area, $A = 2\displaystyle\int_0^2 y\,dx = 2\int_0^2 \frac{x^{3/2}}{\sqrt{2-x}}dx$

Put $x = 2\sin^2\theta$



$$\therefore \quad A = 16\int_0^{\pi/2} \sin^4\theta\, d\theta$$

$$= 16 \times \frac{3 \cdot 1}{4 \cdot 2} \times \frac{\pi}{2} = 3\pi.$$

7. (c): Clearly domain and range of $g(x)$ are $[-1, 1)$ and $\{\pi\}$ respectively.

Since, $\cos^{-1}(\cos x) = -x$ for $x \in [-1, 0]$

$\therefore \quad$ For $-1 \le x \le 0$ the equation becomes

$-x + \pi = 3 \Rightarrow x = \pi - 3 \notin [-1, 0]$

$\therefore \quad$ No solution.

Now, $\cos^{-1}(\cos x) = x$ for $x \in [0, 1]$

For $0 \le x \le 1$ the equation becomes

$x + \pi = 3 \Rightarrow x = 3 - \pi \notin [0, 1]$

$\therefore \quad$ No solution.

8. (c): $\because \dfrac{BD}{DC} = \left|\dfrac{\overrightarrow{AB}}{\overrightarrow{AC}}\right|$; $\quad BD : DC = |\overrightarrow{AB}| : |\overrightarrow{AC}|$

$$\therefore \quad \overrightarrow{AD} = \frac{BD\overrightarrow{AC} + DC\overrightarrow{AB}}{(BD + DC)} = \frac{\dfrac{BD}{DC}\overrightarrow{AC} + \overrightarrow{AB}}{\left(\dfrac{BD}{DC} + 1\right)}$$

$$= \frac{\overrightarrow{AB}|\overrightarrow{AC}| + \overrightarrow{AC}|\overrightarrow{AB}|}{|\overrightarrow{AB}| + |\overrightarrow{AC}|}$$

$$= \frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}\overrightarrow{AB} + \frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}\overrightarrow{AC}$$

$$= \alpha\,\overrightarrow{AB} + \beta\,\overrightarrow{AC} \quad \text{(given)}$$

$$\therefore\quad \alpha = \frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|} \quad \text{and} \quad \beta = \frac{|\overrightarrow{AB}|}{|\overrightarrow{AB}|+|\overrightarrow{AC}|}$$

9. (b): Let $x^3 = u$, then $3x^2dx = du$

Also, $\int f(x)dx = \psi(x)$

Now $\int x^5 f(x^3)dx = \frac{1}{3}\int u\, f(u)du$

$$= \frac{1}{3}\left[u\int f(u)du - \int(\textstyle\int f(u)du)du\right]$$

$$= \frac{1}{3}x^3\psi(x^3) - \int x^2\psi(x^3)dx + C$$

10. (d): Since two parabolas are symmetrical about $y = x$.

Solving $y = x$ and $y^2 - 4x - 8y + 40 = 0$, we get $x^2 - 12x + 40 = 0$ has no real solution.

$\therefore$ They don't intersect.

Point on $(x-4)^2 = 4(y-6)$ is $(6, 7)$ and the corresponding point on $(y-4)^2 = 4(x-6)$ is $(7, 6)$. Minimum distance is $\sqrt{2}$.

11. (d): $\vec{a}\times\vec{r} = \hat{j} \Rightarrow \hat{j}$ is perpendicular to both $\vec{a}$ and $\vec{r}$

$\Rightarrow \quad \vec{a}\cdot\hat{j} = 0$ and $\vec{r}\cdot\hat{j} = 0$

If angle between $\vec{a}$ and $\vec{r}$ is θ then

$\vec{a}\cdot\vec{r} = |\vec{a}||\vec{r}|\cos\theta = ar\cos\theta$

$$= \sqrt{a^2r^2\cos^2\theta} = \sqrt{a^2r^2(1-\sin^2\theta)}$$

$$= \sqrt{a^2r^2 - (\vec{a}\times\vec{r})^2} = \sqrt{(a^2r^2 - 1)}$$

12. (c): $I_{m,n} = \int_0^1 (1+t)^n t^m dt$

Integration by parts gives

$$I_{m,n} = (1+t)^n \left.\frac{t^{m+1}}{m+1}\right|_0^1 - \int_0^1 n(1+t)^{n-1}\frac{t^{m+1}}{m+1}dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1}\int_0^1 t^{m+1}(1+t)^{n-1}dt$$

$$= \frac{2^n}{m+1} - \frac{n}{m+1}I_{m+1,n-1}.$$

13. (c)

14. (b): $y = \dfrac{-2p}{\sqrt{1-p^2}}x + \dfrac{1}{\sqrt{1-p^2}}$

$$\Rightarrow \quad m = -\frac{2p}{\sqrt{1-p^2}} \quad \Rightarrow \quad p^2 = \frac{m^2}{4+m^2}$$

$$\therefore \quad y = mx + \frac{1}{\sqrt{1-\frac{m^2}{4+m^2}}} \Rightarrow y = mx + \sqrt{\frac{4+m^2}{4}}$$

$$\Rightarrow \quad y = mx + \sqrt{1 + \frac{1}{4}m^2}$$

It touches $\dfrac{x^2}{1/4} + \dfrac{y^2}{1} = 1,\ e = \dfrac{\sqrt{3}}{2}$.

15. (d)

16. (c): $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)(3+\cos x)}{x(\tan 4x)}$

$$= \lim_{x\to 0}\frac{1-\cos 2x}{x^2\cdot\left(\frac{\tan 4x}{x}\right)}(3+\cos x)$$

$$= \lim_{x\to 0}\left(\frac{2\sin^2 x}{x^2}\right)\cdot\left(\frac{x}{\tan 4x}\right)\ (3+\cos x) = 2\times\frac{1}{4}\times 4 = 2$$

17. (b)

18. (c): $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)dt$

Differentiating, $\sqrt{1-(f'(x))^2} = f(x)$

$$\Rightarrow \quad 1-(f'(x))^2 = (f(x))^2 \Rightarrow f'(x) = \sqrt{1-(f(x))^2}$$

$$\Rightarrow \quad \frac{dt}{\sqrt{1-f^2}} = dx \Rightarrow \sin^{-1} f = x + c$$

$$\Rightarrow \quad f = \sin(x+c), f(0) = 0 \Rightarrow c = 0 \quad \therefore\ f(x) = \sin x$$

$$f\left(\frac{1}{2}\right) = \sin\frac{1}{2} < \frac{1}{2}, f\left(\frac{1}{3}\right) = \sin\frac{1}{3} < \frac{1}{3}.$$

19. (b): We can have the following distributions

	B_1	B_2	B_3	
Case I	1	1	3	$\rightarrow \dfrac{\lfloor 5}{\lfloor 1\lfloor 1\lfloor 3}\cdot\dfrac{1}{\lfloor 2} = 10$
Case II	2	2	1	$\rightarrow \dfrac{\lfloor 5}{\lfloor 2\lfloor 2\lfloor 1}\cdot\dfrac{1}{\lfloor 2} = 15$

Total number of ways $= 25\times\lfloor 3 = 25\times 6 = 150$

20. (a): Let $P(2r_1, -3r_1, r_1)$ and $Q(3r_2 + 2, -5r_2 + 1, 2r_2 - 2)$ be the points on the given lines so that PQ be the line of shortest distance between the given lines. Now, direction ratios of PQ are

$2r_1 - 3r_2 - 2, -3r_1 + 5r_2 - 1, r_1 - 2r_2 + 2$

since, it is perpendicular to the given lines.

$\therefore\quad 2(2r_1 - 3r_2 - 2) - 3(-3r_1 + 5r_2 - 1) + (r_1 - 2r_2 + 2) = 0$

And $3(2r_1 - 3r_2 - 2) - 5(-3r_1 + 5r_2 - 1) + 2(r_1 - 2r_2 + 2) = 0$

$\Rightarrow\quad 14r_1 - 23r_2 + 1 = 0,\ 23r_1 - 38r_2 + 3 = 0$

$\Rightarrow\quad r_1 = \dfrac{31}{3},\ r_2 = \dfrac{19}{3}$

$\Rightarrow\ P \equiv \left(\dfrac{62}{3}, -31, \dfrac{31}{3}\right)$ and $Q \equiv \left(21, \dfrac{-92}{3}, \dfrac{32}{3}\right)$

and direction ratios of PQ are $\dfrac{1}{3}, \dfrac{1}{3}, \dfrac{1}{3}$

Hence, equation of PQ is

$$\frac{x-21}{1/3} = \frac{y+92/3}{1/3} = \frac{z-32/3}{1/3}$$

$\Rightarrow\quad 3(x - 21) = 3y + 92 = 3z - 32$

21. (a) : At $x = 0$, $f'(0^-) = \lim\limits_{h\to 0} \dfrac{f(0-h) - f(0)}{-h}$

$$= \lim_{h\to 0} \frac{h^2\left|\cos\frac{\pi}{h}\right|}{-h} = -\lim_{h\to 0} h\left|\cos\frac{\pi}{h}\right| = 0$$

$$f'(0^+) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h^2\left(\cos\frac{\pi}{h}\right)}{h} = \lim_{h\to 0} h\left|\cos\frac{\pi}{h}\right| = 0$$

Hence, f is differentiable at $x = 0$ and $f'(0) = 0$

At $x = 2$, $f'(2^-) = \lim\limits_{h\to 0} \dfrac{f(2-h) - f(2)}{-h}$

$$= \lim_{h\to 0} \frac{(2-h)^2\left|\cos\left(\frac{\pi}{2-h}\right)\right|}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2 \sin\left(\frac{\pi}{2} - \frac{\pi}{2-h}\right)}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2}{-h} \sin\left(\frac{\pi}{2} - \frac{\pi}{2-h}\right)$$

$$= \lim_{h\to 0} \frac{(2-h)^2}{-h} \sin\left(-\frac{\pi h}{2(2-h)}\right)$$

$$= \lim_{h\to 0} \frac{(2-h)^2 \sin\left(-\frac{\pi h}{2(2-h)}\right)}{\left(-\frac{\pi h}{2(2-h)}\right)} \cdot \frac{\pi}{2(2-h)} = \pi$$

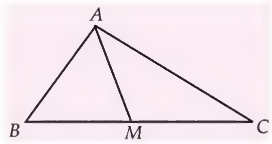
R.H.D. $= f'(2^+) = \lim\limits_{h\to 0} \dfrac{f(2+h) - f(2)}{h}$

$$= \lim_{h\to 0} \frac{(2+h)^2 \sin\left(\frac{\pi}{2} - \frac{\pi}{2+h}\right)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 \sin\left(\frac{\pi h}{2(2+h)}\right)}{\left(\frac{\pi}{2(2+h)}\cdot h\right)} \cdot \frac{\pi}{2(2+h)} = \pi$$

As $f'(2^+) = f'(2^-)$, f is differentiable at $x = 2$

22. (d) : $\overrightarrow{AM} = \dfrac{1}{2}(\overrightarrow{AB} + \overrightarrow{AC})$



$= \dfrac{1}{2}\{(3,\ 0,\ 4) + (5,\ -2,\ 4)\}$

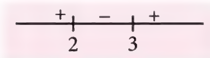
$= \dfrac{1}{2}(8,\ -2,\ 8) = (4,\ -1,\ 4)$

$\therefore\quad |\overrightarrow{AM}| = \sqrt{4^2 + 1^2 + 4^2} = \sqrt{33}$

23. (b) : $f(x) = 2x^3 - 15x^2 + 36x + 1$

We have $f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$

$= 6\,(x - 2)(x - 3)$



$f(0) = 1$

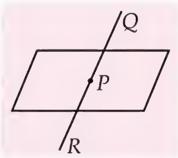
$f(3) = 54 - 135 + 108 + 1 = 28$

$f(2) = 16 - 60 + 72 + 1 = 29$

Then the range $= [1, 29]$

Hence, the given function is onto. But it is not one-one, as f' takes both positive and negative values.

24. (a) : Let equation of line QR be



$$\frac{x-2}{1} = \frac{y-3}{4} = \frac{z-5}{1}$$

Let the point P be given as

$P(2 + t, 3 + 4t, 5 + t)$

As P lies in the plane $5x - 4y - z = 1$

$\therefore\quad 5(2 + t) - 4(3 + 4t) - (5 + t) = 1$

$\Rightarrow\quad 10 + 5t - 12 - 16t - 5 - t = 1$

$\Rightarrow\quad -12t - 7 = 1$

$\Rightarrow\quad 12t = -8\ \ \therefore\ \ t = -\dfrac{8}{12} = -\dfrac{2}{3}$

The co-ordinates of P are $\left(\dfrac{4}{3}, \dfrac{1}{3}, \dfrac{13}{3}\right)$

Again let $S \equiv (2 + m, 3 + 4m, 5 + m)$ for being a parameter

As S is the foot of perpendicular, we have

$1(2 + m - 2) + 4(3 + 4m - 1) + 1(5 + m - 4) = 0$

$\Rightarrow\quad m + 4(4m + 2) + m + 1 = 0$

$\Rightarrow\quad 18m + 9 = 0\ \therefore\ \ m = -\dfrac{1}{2}$

Thus $S \equiv \left(\dfrac{3}{2}, 1, \dfrac{9}{2}\right)$

We have, by distance formula

$$PS=\sqrt{\left(\frac{3}{2}-\frac{4}{3}\right)^2+\left(1-\frac{1}{3}\right)^2+\left(\frac{9}{2}-\frac{13}{3}\right)^2}$$

$$=\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{2}{3}\right)^2+\left(\frac{1}{6}\right)^2}=\sqrt{\frac{1}{36}+\frac{4}{9}+\frac{1}{36}}=\sqrt{\frac{9}{18}}=\frac{1}{\sqrt{2}}$$

25. (c) : $|\cos x|+\cos x>0 \Rightarrow x\in\left[0,\frac{\pi}{2}\right)$

For $x\in\left[0,\frac{\pi}{2}\right)$, $1\le\sin x+\cos x\le\sqrt{2}$

But $\sin x+\cos x\neq 1 \Rightarrow x\in\left(0,\frac{\pi}{2}\right)$

Now, for $x\in\left(0,\frac{\pi}{2}\right)$

$\log_{\sin x+\cos x}(|\cos x|+\cos x)\ge 0$

$\Rightarrow \cos x\ge\frac{1}{2} \Rightarrow x\in\left(0,\frac{\pi}{3}\right]$.

26. (a) : 1st solution : (Direct counting) Let D_4 shows, say, 6

Then there are three ways in which one of D_1, D_2 and D_3 shows the same number

(i) Exactly one of D_2's show the same number

Number of ways = $({}^3C_1).(5)(5) = 75$

(ii) Exactly two of D_2's show the same number

Number of ways = $({}^3C_2)(5) = 15$

(iii) All three D_2's show the same number = 1

Now, favourable outcomes

= 75 + 15 + 1 = 91

$\therefore$ Desired probability $=\frac{{}^6C_1\cdot 91}{6^4}=\frac{91}{216}$

2nd solution : (complementary counting)

The number of ways in which one of D_1, D_2 and D_3 shows the same number as D_4 = the number of total ways – number of ways in which D_1, D_2, D_3 don't show a number appearing on $D_4 = 6^4 - {}^6C_1\cdot 5^3$

$\therefore$ Required probability $=\frac{6^4-6\cdot 5^3}{6^4}=1-\left(\frac{5}{6}\right)^3=\frac{91}{216}$

27. (d) : Let $Q=\begin{vmatrix}2^2a_{11} & 2^3a_{12} & 2^4a_{13}\\ 2^3a_{21} & 2^4a_{22} & 2^5a_{23}\\ 2^4a_{31} & 2^5a_{32} & 2^6a_{33}\end{vmatrix}$

$$=2^9\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ 2a_{21} & 2a_{22} & 2a_{23}\\ 2^2a_{31} & 2^2a_{32} & 2^2a_{33}\end{vmatrix}=2^9\cdot 2\cdot 2^2\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$$

$= 2^{12}\det P = 2^{12}\cdot 2 = 2^{13}$

28. (b) : Since, $\frac{1+z}{1+\bar{z}}=\frac{1+z}{1+\frac{1}{z}}=z$

Then, the arg of the number $\frac{1+z}{1+\bar{z}}$ is just the argument of z and that's θ.

29. (b) : 1st solution : Let $1 + a = b$

$(b^{1/3}-1)x^2+(b^{1/2}-1)x+b^{1/6}-1=0$

$$\Rightarrow \left(\frac{b^{1/3}-1}{b-1}\right)x^2+\left(\frac{b^{1/2}-1}{b-1}\right)x+\frac{b^{1/6}-1}{b-1}=0$$

Taking limit on both sides we have $\frac{1}{3}x^2+\frac{1}{2}x+\frac{1}{6}=0$

$\Rightarrow 2x^2+3x+1=0 \quad \therefore \quad x=-1,-\frac{1}{2}$.

2nd solution :

$$\alpha+\beta=\frac{1-\sqrt{1+a}}{\sqrt[3]{1+a}-1},\ \alpha\beta=\frac{\sqrt[6]{1+a}-1}{\sqrt[3]{1+a}-1}$$

$$\lim_{a\to 0}(\alpha+\beta)=\frac{-1/2}{1/3}=-\frac{3}{2}$$

$$\lim_{a\to 0}\alpha\beta=\frac{1/6}{1/3}=\frac{1}{2}$$

Thus $\alpha,\beta\in\left\{-1,\frac{-1}{2}\right\}$

30. (b) : $3y^2+4y=6x-8$

$$\Rightarrow 3\left(y^2+\frac{4}{3}y\right)=6x-8\Rightarrow\left(y+\frac{2}{3}\right)^2=2x-\frac{8}{3}+\frac{4}{9}$$

$$\Rightarrow\left(y+\frac{2}{3}\right)^2=2\left(x-\frac{10}{9}\right)$$

Let any point on the axis be $\left(a,-\frac{2}{3}\right)$.

$$y+\frac{2}{3}=m\left(x-\frac{10}{9}\right)-m-\frac{1}{2}m^3$$

$$\Rightarrow 0=m\left[a-\frac{10}{9}-1-\frac{1}{2}m^2\right]$$

$$\Rightarrow \quad a-\frac{19}{9}=\frac{1}{2}m^2\Rightarrow m^2=2\left(a-\frac{19}{9}\right)$$

$\therefore\ a>\frac{19}{9}$.

◈◈

JEE MAIN

PRACTICE PAPER 2018

*ALOK KUMAR, B.Tech, IIT Kanpur

1. If $2\tan^2x - 5\sec x = 1$ for exactly 7 distinct values of $x \in \left[0, \frac{n\pi}{2}\right], n \in N$ then the greatest value of n is
(a) 13 (b) 17 (c) 19 (d) 15

2. Let $\theta \in [0, 4\pi]$ satisfying the equation $(\sin\theta + 2)(\sin\theta + 3)(\sin\theta + 4) = 6$. If the sum of all values of θ is $K\pi$ then value of K is
(a) 6 (b) 5 (c) 4 (d) 2

3. The number of solutions of the equation $16(\sin^5x + \cos^5x) = 11(\sin x + \cos x)$ in the interval $[0, 2\pi]$ is
(a) 6 (b) 7 (c) 8 (d) 9

4. The equation $2x = (2n + 1)\pi(1 - \cos x)$, (where n is a positive integer)
(a) has infinitely many real roots
(b) has exactly one real root
(c) has exactly $2n + 2$ real roots
(d) has exactly $2n + 3$ real roots

5. Number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is
(a) 0 (b) 1 (c) 2 (d) 3

6. $|x^2 \sin x + \cos^2 x e^x + \ln^2 x| < x^2|\sin x| + \cos^2 x e^x + \ln^2 x$ true for $x \in$
(a) $(-\pi, 0)$ (b) $\left(0, \frac{\pi}{2}\right)$
(c) $\left(\frac{\pi}{2}, \pi\right)$ (d) $(2n\pi, (2n + 1)\pi)\ n \in N$

7. If $\log_{0.5}\sin x = 1 - \log_{0.5}\cos x$ then number of values of $x \in [-2\pi, 2\pi]$ is
(a) 1 (b) 2 (c) 3 (d) 4

8. If $2^{\sqrt{\sin^2 x - 2\sin x + 5}}\frac{1}{4^{\sin^2 y}} \le 1$, then the ordered pair (x, y) is equal to $(m, n \in I)$
(a) $x = (4n+1)\frac{\pi}{2}, y = (2m+1)\frac{\pi}{2}$
(b) $x = 2n\pi, y = 2m\pi$
(c) $x = (2n+1)\frac{\pi}{2}, y = (2m+1)\frac{\pi}{2}$
(d) $x = n\pi, y = m\pi$

9. The value of $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50° =$
(a) $\frac{4}{5}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) 3

10. Number of ordered pairs (a, x) satisfying the equation $\sec^2(a + 2)x + a^2 - 1 = 0$; $-\pi < x < \pi$ is
(a) 1 (b) 2 (c) 3 (d) 5

11. If $a\sin^2x + b\cos^2x = c$, $b\sin^2y + a\cos^2y = d$ and $a\tan x = b\tan y$ then $\frac{a^2}{b^2} =$
(a) $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ (b) $\frac{(a+d)(c+a)}{(b+c)(d+b)}$
(c) $\frac{(a-d)(b-a)}{(a-c)(c-b)}$ (d) $\frac{(d-a)(c-a)}{(b-c)(d-b)}$

12. If $\cos\theta = \frac{a\cos\phi + b}{a + b\cos\phi}$ then $\tan\frac{\theta}{2}$ is equal to
(a) $\sqrt{\left(\frac{a-b}{a+b}\right)}\tan(\phi/2)$ (b) $\sqrt{\left(\frac{a+b}{a-b}\right)}\cos(\phi/2)$
(c) $\sqrt{\left(\frac{a-b}{a+b}\right)}\sin(\phi/2)$ (d) none of these

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91). He trains IIT and Olympiad aspirants.

13. If α is the angle in which each side of a regular polygon of n sides subtends at its centre then $1 + \cos\alpha + \cos2\alpha + \cos3\alpha + ... + \cos(n-1)\alpha$ is equal to

(a) n (b) 0 (c) 1 (d) $n-1$

14. If in a triangle $\frac{s-a}{11}=\frac{s-b}{12}=\frac{s-c}{13}$ and $\lambda\tan^2(A/2) = 455$, then λ must be

(a) 1155 (b) 1551 (c) 5511 (d) 1515

15. If $2\sin x - \cos2x = 1$, then $\cos^2x + \cos^4x$ is equal to

(a) 1 (b) -1 (c) $-\sqrt{5}$ (d) $\sqrt{5}$

16. A set of values of x, satisfying the equation $\cos^2\left(\frac{1}{2}px\right)+\cos^2\left(\frac{1}{2}qx\right)=1$ form an arithmetic progression with common difference

(a) $\frac{2}{p+q}$ (b) $\frac{2}{p-q}$

(c) $\frac{\pi}{p+q}$ (d) none of these

17. If $\cos^6\alpha + \sin^6\alpha + k\sin^2 2\alpha = 1\ \forall\ \alpha \in (0, \pi/2)$, then k is

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

18. The most general solution of the equations $\tan\theta = -1,\ \cos\theta = \frac{1}{\sqrt{2}}$ is

(a) $n\pi + 7\pi/4$ (b) $n\pi+(-1)^n\frac{7\pi}{4}$

(c) $2n\pi+\frac{7\pi}{4}$ (d) none of these

19. The least positive values of x satisfying the equation $8^{1+|\cos x|+\cos^2 x+|\cos^3 x|+...\infty} = 4^3$ will be (where $|\cos x| < 1$)

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$ (d) none of these

20. For each real number x such that $-1 < x < 1$, let $A(x)$ be the matrix $(1-x)^{-1}\begin{bmatrix}1 & -x\\ -x & 1\end{bmatrix}$ and $z=\frac{x+y}{1+xy}$. Then,

(a) $A(z) = A(x) + A(y)$ (b) $A(z) = A(x)\,[A(y)]^{-1}$

(c) $A(z) = A(x)\,A(y)$ (d) $A(z) = A(x) - A(y)$

21. If A is a square matrix of order 3 such that $|A| = 2$ then $|(\text{adj } A^{-1})^{-1}|$ is

(a) 1 (b) 2 (c) 4 (d) 8

22. If A and B are square matrices of the same order and A is non-singular, then for a positive integer n, $(A^{-1}BA)^n$ is equal to

(a) $A^{-n}B^nA^n$ (b) $A^nB^nA^{-n}$

(c) $A^{-1}B^nA$ (d) $n(A^{-1}BA)$

23. If $A=\begin{bmatrix} i & -i\\ -i & i\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$, then A^8 equals

(a) $4B$ (b) $128B$ (c) $-128B$ (d) $-64B$

24. If $p + q + r = 0$ and $\begin{vmatrix} pa & qb & rc\\ qc & ra & pb\\ rb & pc & qa\end{vmatrix}=k\begin{vmatrix} a & b & c\\ c & a & b\\ b & c & a\end{vmatrix}$, then $k =$

(a) 0 (b) abc (c) pqr (d) $a+b+c$

25. A square matrix P satisfies $P^2 = I - P$, where I is an identity matrix of order as order of P. If $P^n = 5I - 8P$, then $n =$

(a) 4 (b) 5 (c) 6 (d) 7

26. The digits A, B, C are such that the three digit numbers $A88$, $6B8$, $86C$ are divisible by 72, then the determinant $\begin{vmatrix} A & 6 & 8\\ 8 & B & 6\\ 8 & 8 & C\end{vmatrix}$ is divisible by

(a) 76 (b) 144 (c) 216 (d) 276

27. Let A, B be square matrix such that $AB = 0$ and B is non singular then

(a) $|A|$ must be zero but A may non zero

(b) A must be zero matrix

(c) nothing can be said in general about A

(d) none of these

28. Let $x > 0$, $y > 0$, $z > 0$ are respectively the 2nd, 3rd, 4th terms of a G.P and

$$\Delta=\begin{vmatrix} x^k & x^{k+1} & x^{k+2}\\ y^k & y^{k+1} & y^{k+2}\\ z^k & z^{k+1} & z^{k+2}\end{vmatrix}=(r-1)^2\left(1-\frac{1}{r^2}\right)$$

(where r is the common ratio) then

(a) $k=-1$ (b) $k = 1$

(c) $k = 0$ (d) None of these

29. $A = [a_{ij}]_{m\times n}$ and $a_{ij} = i^2 - j^2$ then A is necessarily

(a) a unit matrix

(b) symmetric matrix

(c) skew symmetric matrix

(d) zero matrix

30. If $A=\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$ then $A^{16}=$

(a) $\begin{bmatrix}0 & 256\\ 256 & 0\end{bmatrix}$ (b) $\begin{bmatrix}256 & 0\\ 0 & 256\end{bmatrix}$

(c) $\begin{bmatrix}-16 & 0\\ 0 & -16\end{bmatrix}$ (d) $\begin{bmatrix}0 & 16\\ 16 & 0\end{bmatrix}$

31. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}, a\, b \in N$, then number of matrix 'B' such that $AB = BA$ are

(a) 0 (b) 1

(c) finitely many (d) infinite

32. Let A and B are two non-singular square matrices, and A^T and B^T are the transpose matrices of A and B respectively, then which of the following is correct?

(a) B^TAB is symmetric matrix if and only if A is symmetric

(b) B^TAB is symmetric matrix if and only if B is symmetric

(c) B^TAB is skew symmetric matrix for every matrix A

(d) B^TAB is skew symmetric matrix if B is skew symmetric

33. $|A_{3\times 3}| = 3$, $|B_{3\times 3}| = -1$, and $|C_{2\times 2}| = 2$, then $|2ABC| =$

(a) $2^3(6)$ (b) $2^3(-6)$

(c) $2(-6)$ (d) none of these

34. If A and B are two matrices such that $AB = B$ and $BA = A$, then

(a) $(A^6 - B^5)^3 = A - B$ (b) $(A^5 - B^5)^3 = A^3 - B^3$

(c) $A - B$ is idempotent (d) $A - B$ is nilpotent

35. Let $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $B=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ are two matrices such that $AB = BA$ and $c \neq 0$, then value of $\dfrac{a-d}{3b-c}$ is:

(a) 0 (b) 2 (c) −2 (d) −1

36. Let $f(\alpha)=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, then $(f(\alpha))^{-1}$ is equal to

(a) $f(\alpha)$ (b) $f(-\alpha)$

(c) $f(\alpha - 1)$ (d) none of these

37. If $A=\begin{bmatrix}5 & -6\\ 1 & -1\end{bmatrix}$ then determinant of $A^{1003} - 5A^{1002}$ is

(a) 1 (b) 2 (c) 4 (d) 6

38. $f: R \to R, f(x) = x|x|$ is

(a) one-one but not onto

(b) onto but not one-one

(c) Both one-one and onto

(d) neither one-one nor onto

39. The domain of the function

$f(x)=\sin^{-1}\left(\log_2\left(\dfrac{x^2}{2}\right)\right)$ is

(a) $[-2, 2]$ (b) $[-2, -1]$

(c) $[1, 2]$ (d) $[-2, -1] \cup [1, 2]$

40. The range of $f(x)=\dfrac{3}{5+4\sin 3x}$ is

(a) $\left[\dfrac{1}{3}, 3\right]$ (b) $\left[\dfrac{1}{3}, 1\right]$

(c) $[1, 3]$ (d) $\left(-\infty, \dfrac{1}{3}\right) \cup (3, \infty)$

41. The range of $\dfrac{x^2-x+1}{x^2+x+1}$ is

(a) $\left[\dfrac{1}{3}, 3\right]$ (b) $\left[\dfrac{1}{3}, 1\right]$

(c) $[1, 3]$ (d) $(-\infty, \dfrac{1}{3}] \cup [3, \infty)$

42. Domain of the function $f(x)=\sqrt{5|x|-x^2-6}$ is

(a) $(-\infty, 2) \cup (3, \infty)$ (b) $[-3, -2] \cup [2, 3]$

(c) $(-\infty, -2) \cup (2, 3)$ (d) $R - \{-3, -2, 2, 3\}$

43. The range of the function

$f(x)=\cos^2\dfrac{x}{4}+\sin\dfrac{x}{4},\ x \in R$ is

(a) $\left[0, \dfrac{5}{4}\right]$ (b) $\left[1, \dfrac{5}{4}\right]$

(c) $\left(-1, \dfrac{5}{4}\right)$ (d) $\left[-1, \dfrac{5}{4}\right]$

44. Range of the function $f(x)=x^2+\dfrac{1}{x^2+1}$, is

(a) $[1, \infty]$ (b) $[2, \infty)$

(c) $\left[\dfrac{3}{2}, \infty\right)$ (d) $(-\infty, \infty)$

45. The inverse of $f(x) = (5 - (x - 8)^5)^{1/3}$ is

(a) $5 - (x - 8)^5$ (b) $8 + (5 - x^3)^{1/5}$

(c) $8 - (5 - x^3)^{1/5}$ (d) $(5 - (x - 8)^{1/5})^3$

46. Minimum value of function $f(x) = x^3(x^3+1)(x^3+2)(x^3+3) : x \in R$, is

(a) –2 (b) –1 (c) 1 (d) none

47. The domain of the function $f(x) = \log_{10}\{1 - \log_{10}(x^2 - 5x + 10)\}$ is

(a) $(0, \infty)$ (b) $(0, 5)$

(c) $(-\infty, 0)$ (d) None of these

48. The range of the function

$f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$ where [.] = G. I. F

(a) $\{\pi\}$ (b) $\left\{\frac{\pi}{2}\right\}$ (c) $\{2\pi\}$ (d) $\{0\}$

49. The domain of definition of the function, $f(x)$ given by the equation $2^x + 2^y = 2$ is

(a) $0 < x \le 1$ (b) $0 \le x \le 1$

(c) $-\infty < x \le 0$ (d) $-\infty < x < 1$

50. If $f : R \to R$ is a function satisfying the property $f(x+1) + f(x+3) = 2$ for all $x \in R$ then f is

(a) periodic with period 3

(b) periodic with period 4

(c) non periodic

(d) periodic with period 5

51. Let $f : R \to R - \{3\}$ be a function such that for some $p > 0$, $f(x+p) = \frac{f(x)-5}{f(x)-3}$ for all $x \in R$. Then, period of f is

(a) $2p$ (b) $3p$ (c) $4p$ (d) $5p$

52. The period of the function $f(x) = (-1)^{[x]}$ where [.] = G.I.F

(a) 2 (b) 1 (c) 3 (d) 4

53. If $f(x) = x - \frac{1}{x}$, then number of solutions of $f(f(f(x))) = 1$ is

(a) 1 (b) 4 (c) 6 (d) 2

54. If $f(x) = x(x-1)$ is a function from $\left[\frac{1}{2}, \infty\right)$ to $\left[-\frac{1}{4}, \infty\right)$, then $\{x \in R : f^{-1}(x) = f(x)\}$ is

(a) null set

(b) $\{1\}$

(c) $\{0, 2\}$

(d) a set containing 3 elements

55. Let $f : \left[\frac{-\pi}{3}, \frac{2\pi}{3}\right] \to [0, 4]$ be a function defined as $f(x) = \sqrt{3}\sin x - \cos x + 2$ then $f^{-1}(x)$ is given by

(a) $\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{\pi}{6}$ (b) $\sin^{-1}\left(\frac{x+2}{2}\right) + \frac{\pi}{6}$

(c) $\frac{2\pi}{3} - \cos^{-1}\left(\frac{x-2}{2}\right)$ (d) Does not exist

56. The value of the parameter α, for which the function $f(x) = 1 + \alpha x$, $\alpha \ne 0$, is the inverse of itself, is

(a) –2 (b) –1 (c) 1 (d) 2

57. If for nonzero x, $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$ then $f(x^2) =$

(a) $\frac{3 + 2x^4 - x^2}{5x^2}$ (b) $\frac{3 - 2x^4 + x^2}{5x^2}$

(c) $\frac{3 - 2x^4 - x^2}{5x^2}$ (d) $\frac{3 + 2x^4 + x^2}{5x^2}$

58. If $g(x)$ is a polynomial satisfying $g(x)\,g(y) = g(x) + g(y) + g(xy) - 2$ for all real x and y and $g(2) = 5$, then $g(3)$ is equal to

(a) 10 (b) 24 (c) 21 (d) 15

59. Let $f\left(\frac{x+y}{2}\right) = \frac{1}{2}(f(x) + f(y))$ for real x and y. If $f'(x)$ exists and equals to –1 and $f(0) = 1$, then the value of $f(2)$ is

(a) 1 (b) –1 (c) $\frac{1}{2}$ (d) 2

60. A function $f : R \to R$ satisfies the equation $f(x)\,f(y) - f(xy) = x + y \;\; \forall\, x, y \in R$ and $f(1) > 0$, then

(a) $f(x)f^{-1}(x) = x^2 - 4$ (b) $f(x)f^{-1}(x) = x^2 - 6$

(c) $f(x)f^{-1}(x) = x^2 - 1$ (d) none of these

SOLUTIONS

1. **(d) :** $\sec x = 3 \Rightarrow \cos x = \frac{1}{3}$

which gives two values of x in each of $[0, 2\pi]$, $(2\pi, 4\pi]$, $(4\pi, 6\pi]$ and one value in $6\pi + \frac{3\pi}{2} = 15\frac{\pi}{2}$

$\therefore$ Greatest value of $n = 15$

2. **(b) :** $\sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2}, \frac{7\pi}{2}$

$\therefore$ $K = 5$

3. **(a) :** $16(\sin^5 x + \cos^5 x) - 11(\sin x + \cos x) = 0$

$\Rightarrow$ $(\sin + \cos x)\{16(\sin^4 x - \sin^3 x\cos x + \sin^2 x\cos^2 x - \sin x\cos^3 x + \cos^4 x) - 11\} = 0$

$\Rightarrow$ (sin$x$ + cos$x$){16(1 – sin^2x cos^2x – sin$x$ cos$x$) – 11}= 0

$\Rightarrow$ (sin$x$ + cos$x$)(4sin$x$ cos$x$ – 1)(4sin$x$ cos$x$ + 5) = 0

As 4sin$x$ cos$x$ + 5 ≠ 0, we have

sin$x$ + cos$x$ = 0 or 4sin$x$ cos$x$ – 1 = 0

The required values are π/12, 5π/12, 9π/12, 13π/12, 17π/ 12, 21π/12, – they are 6 solutions on [0, 2π].

4. (c) : $\sin^2\left(\frac{x}{2}\right)=\frac{x}{(2n+1)\pi}$ the graph of $\sin^2\left(\frac{x}{2}\right)$ will be above the x-axis and will be meeting the x-axis at 0, 2π, 4π, . . . etc. It will attain maximum values at odd multiples of π *i.e.*, π, 3π, . . . (2n + 1)π. The last point after which graph of $y=\frac{x}{(2n+1)\pi}$ will stop cutting will be (2n + 1)π.

Total intersection = 2(n + 1)

5. (c) : Given equation is $\frac{1+\sin x}{\cos x}=2\cos x$

$\Rightarrow$ 1 + sin$x$ = 2 cos^2x = 2(1 – sin^2x)

$\Rightarrow$ 2 sin^2x + sin$x$ – 1 = 0

$\Rightarrow$ (1 + sin$x$)(2 sin$x$ – 1) = 0

$\Rightarrow$ sin x = –1 or 1/2

Now, sin x = –1 $\Rightarrow$ tan$x$ and sec$x$ not defined.

sin$x$ = 1/2 $\Rightarrow$ x = π/6 or 5π/6.

∴ The required number of solution is 2.

6. (a) : $|a + b + c| < |a| + |b| + |c|$

If a, b, c do not have same sign.

$\Rightarrow x^2\sin x < 0$ ∴ $x \in (-\pi, 0)$

7. (d) : Given, $\log_{0.5} \sin x = 1 - \log_{0.5} \cos x$, $x \in [-2\pi, 2\pi]$

sin$x$ > 0 and cos$x$ > 0

$\sin x\cos x=\frac{1}{2}$

sin2x = 1, 2x ∈ [–4π, 4π]

$\Rightarrow$ 4 solutions

8. (c) : ∵ sin^2x – 2sin$x$ + 5 = (sin$x$ – 1)^2 + 4 ≥ 4

$\therefore\ 2^{\sqrt{\sin^2 x-2\sin x+5}}\ge 2^2=4$

and $\sin^2 y\le 1 \Rightarrow \frac{1}{4^{\sin^2 y}}\ge\frac{1}{4}$

∴ L.H.S. ≥ 1 and according to question L.H.S. ≤ 1 therefore, L.H.S. = 1

for which sin^2x – 2 sin$x$ + 5 = 4

$\Rightarrow$ (sin$x$ – 1)^2 = 0

$\Rightarrow$ sin$x$ = 1 $\Rightarrow$ $x = (2n+1)\frac{\pi}{2}$

and sin^2y = 1 or cos$y$ = 0

$\Rightarrow$ $y = (2m+1)\frac{\pi}{2}$

9. (c) : cos^{2}10° – cos10° cos50° + cos^{2}50°

$$=\frac{1}{2}\left[1+\cos 20^\circ-\left(\cos 60^\circ+\cos 40^\circ\right)+\left(1+\cos 100^\circ\right)\right]$$

$$=\frac{1}{2}\left[1+\cos 20^\circ-\frac{1}{2}-\cos 40^\circ+1-\cos 80^\circ\right]$$

$$=\frac{1}{2}\left[\frac{3}{2}+\cos 20^\circ-\left(2\cos 60^\circ\cos 20^\circ\right)\right]=\frac{3}{4}$$

10. (c) : Given equation is sec$^2(a + 2)x + a^2 - 1 = 0$

$\Rightarrow$ tan$^2(a + 2)x + a^2 = 0$

$\Rightarrow$ tan$^2(a + 2)x = 0$ and $a = 0$

$\Rightarrow \tan^2 2x=0 \Rightarrow x=0,\frac{\pi}{2},\frac{-\pi}{2}$

∴ (0, 0), (0, π/2), (0, –π/2) are ordered pairs satisfying the equation.

11. (a) : $a\tan^2x + b = c(1 + \tan^2x)$

$\Rightarrow \tan^2 x=\left(\frac{c-b}{a-c}\right), \tan^2 y=\left(\frac{d-a}{b-d}\right)$

$\therefore\ \frac{a^2}{b^2}=\frac{\tan^2 y}{\tan^2 x}=\frac{(a-d)(c-a)}{(b-c)(d-b)}$

12. (a) : $\tan\theta/2=\sqrt{\left(\frac{1-\cos\theta}{1+\cos\theta}\right)}=\sqrt{\frac{1-\left(\frac{a\cos\phi+b}{a+b\cos\phi}\right)}{1+\left(\frac{a\cos\phi+b}{a+b\cos\phi}\right)}}$

$=\sqrt{\frac{(a-b)(1-\cos\phi)}{(a+b)(1+\cos\phi)}}=\sqrt{\frac{(a-b)}{(a+b)}}\tan(\phi/2)$

13. (b) : cosα + cos(α + β) + ... + cos(α + (n – 1)β)

$$=\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\cos\left(\alpha+\frac{(n-1)\beta}{2}\right)$$

14. (a) : $\frac{s-a}{11}=\frac{s-b}{12}=\frac{s-c}{13}=\frac{s}{36}$

On solving we get, $\tan^2(A/2)=\frac{13}{33}$ $\Rightarrow$ λ = 1155

15. (a) : Given, 2 sin$x$ + 2sin^2x – 1 = 1

or sin^2x + sin$x$ – 1 = 0

$\therefore\ \sin x=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1+\sqrt{5}}{2}$

$$\Rightarrow \sin^2 x = \frac{3-\sqrt{5}}{2} \Rightarrow \cos^2 x = \frac{\sqrt{5}-1}{2}$$

$$\therefore \cos^2 x(1+\cos^2 x) = \frac{\sqrt{5}-1}{2} \times \frac{\sqrt{5}+1}{2} = 1$$

16. (d) : $1 + \cos px + 1 + \cos qx = 2$

$$\Rightarrow \cos\left(\frac{p+q}{2}\right)x \cos\left(\frac{p-q}{2}\right)x = 0$$

$$\Rightarrow x = \frac{(2n+1)\pi}{p+q} \text{ or } \frac{(2n+1)\pi}{p-q}$$

for $n = 0, \pm 1, \pm 2, ...$

forms an A.P. with common difference $\frac{2\pi}{p+q}$ or $\frac{2\pi}{p-q}$

17. (a) : The given condition can be written as $(\cos^2\alpha + \sin^2\alpha)^3 - 3\sin^2\alpha \cos^2\alpha(\cos^2\alpha + \sin^2\alpha) + k\sin^2 2\alpha = 1$

$$\Rightarrow \left(-\frac{3}{4}\right)\sin^2 2\alpha + k\sin^2 2\alpha = 0$$

Showing that $k = \frac{3}{4}$.

18. (c) : We have, $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$

The value of θ lying between $\frac{3\pi}{2}$ and 2π and satisfying these two is $\frac{7\pi}{4}$. Therefore the most general solution is $\theta = 2n\pi + 7\pi/4$ where $n \in Z$

19. (a) : $\because 1 + |\cos x| + \cos^2 x.... = \frac{1}{1-|\cos x|}$

$$\Rightarrow \frac{1}{8^{1-|\cos x|}} = 4^3 \Rightarrow 2^{\frac{3}{1-|\cos x|}} = 2^6$$

$$\Rightarrow \frac{3}{1-|\cos x|} = 6 \Rightarrow 1-|\cos x| = \frac{1}{2}$$

$$|\cos x| = \frac{1}{2} \Rightarrow \cos x = \pm\frac{1}{2}$$

For least positive value of x, $x = \frac{\pi}{3}$

20. (c) : $A(z) = A\left(\frac{x+y}{1+xy}\right)$

$$= \left[\frac{1+xy}{(1-x)(1-y)}\right]\begin{bmatrix} 1 & -\left(\frac{x+y}{1+xy}\right) \\ -\left(\frac{x+y}{1+xy}\right) & 1 \end{bmatrix}$$

$\therefore \quad A(x).A(y) = A(z)$

21. (c)

22. (c) : $(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA) = A^{-1}B(AA^{-1})BA$
$= A^{-1}BIBA = A^{-1}B^2A$

$\Rightarrow (A^{-1}BA)^3 = (A^{-1}B^2A)^2 = A^{-1}B^2(AA^{-1})BA$
$= A^{-1}B^2IBA = A^{-1}B^3A$ and so on

$\Rightarrow (A^{-1}BA)^n = A^{-1}B^nA$

23. (b) : We have, $A = iB$

$$\Rightarrow A^2 = (iB)^2 = i^2B^2 = -B^2 = -\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2B$$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow (A^4)^2 = (8B)^2 \Rightarrow A^8 = 64B^2 = 128B$

24. (c) : $p + q + r = 0$

$\Rightarrow p^3 + q^3 + r^3 = 3pqr$

$$\text{Now, } \begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr(a^3 + b^3 + c^3 - 3abc)$$

$$= pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \Rightarrow k = pqr$$

25. (c) : Since $P^2 = I - P$ (given) ...(i)

$\Rightarrow P^3 = P(I - P)$

$\Rightarrow P - P^2 = P - (I - P)$ (using(i))

$\therefore P^3 = 2P - I$ ...(ii)

Similarly, $P^4 = 2P^2 - P = 2I - 3P$ and $P^5 = 5P - 3I$ and $P^6 = 5P^2 - 3P = 5I - 8P$

26. (b) : $100A + 80 + 8 = 72\lambda_1$

$600 + 10B + 8 = 72\lambda_2$

$800 + 60 + C = 72\lambda_3$, $\lambda_1, \lambda_2, \lambda_3 \in I$

$$\text{Let } \Delta = \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$$

$$= \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 72\lambda_1 & 72\lambda_2 & 72\lambda_3 \end{vmatrix} (R_3 \leftrightarrow R_3 + 10R_2 + 100R_1) \quad ...(i)$$

Now, $A88$ is divisible by 72

$\Rightarrow A88$ is divisible by 9

$\therefore A = 2$

Also, $6B8$ is divisible by 9

Substituting these values in (i) we get Δ is divisible by $144 \Rightarrow B = 4$

27. (b) : $A.B = 0 \Rightarrow A.B.B^{-1} = 0.B^{-1}$

$\Rightarrow A.I = 0 \Rightarrow A = 0$

28. (a) : $x^k y^k z^k \begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^3r^6 \end{vmatrix}$

$a^{3(k+1)}.\ r^{3(2k+1)}[r-1](r^4-1)-(r^2-1)^2] \Rightarrow k=-1$

29. (c) : $a_{ji} = j^2 - i^2 = -(i^2 - j^2) = -a_{ij}$

30. (b) : $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}, A^4 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$

$A^8 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}, A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

31. (d) : $AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}, BA = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

$AB = BA \Rightarrow a = b$

32. (a) : $(B^TAB)^T = B^TA^T(B^T)^T = B^TA^TB$
$= B^TAB$ iff A is symmetric
$\therefore$ B^TAB is symmetric iff A is symmetric
Also, $(B^TAB)^T = B^TA^TB = (-B)A^TB$
$\therefore$ B^TAB is not skew symmetric if B is skew symmetric

33. (d) : $2ABC$ is not defined
$\therefore$ there is no solution

34. (d) : Since $AB = B$ and $BA = A$
$\therefore$ A and B both are idempotent
$(A-B)^2 = A^2 - AB - BA + B^2 = A - B - A + B = 0$
$\therefore$ $A - B$ is nilpotent

35. (d) : $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$

$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$

If $AB = BA$, then $a + 2c = a + 3b$
$\Rightarrow$ $2c = 3b \Rightarrow b \neq 0$
Now, $b + 2d = 2a + 4b$
$\Rightarrow$ $2a - 2d = -3b$

$$\therefore \quad \frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b-\frac{3}{2}b} = -1$$

36. (b) : $(f(\alpha))^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-\alpha)$

37. (d) : $|A^{1003} - 5A^{1002}| = |A^{1002}(A-5I)|$
$= |A^{1002}|\ |A-5I| = |A|^{1002}\ |A-5I|$

$= 1 \times \begin{vmatrix} 0 & -6 \\ 1 & -6 \end{vmatrix} = 6$

38. (c) : Given that $f(x) = \begin{cases} x^2 \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x^2 \text{ if } x < 0 \end{cases}$

39. (d) : $f(x) = \sin^{-1}\left(\log_2\left(\frac{x^2}{2}\right)\right) \in R$

$\Leftrightarrow -1 \le \log_2\left(\frac{x^2}{2}\right) \le 1 \Leftrightarrow \frac{1}{2} \le \frac{x^2}{2} \le 2$

$\Leftrightarrow 1 \le x^2 \le 4 \quad \Leftrightarrow x \in [-2,-1] \cup [1,2]$

40. (a) : $-1 \le \sin 3x \le 1$

41. (a) : Let $y = \frac{x^2-x+1}{x^2+x+1}$

$\Rightarrow$ $yx^2 + yx + y = x^2 - x + 1$
$\Rightarrow$ $(y-1)x^2 + (y+1)x + (y-1) = 0$
Now, $x \in R \Rightarrow$ Discriminant ≥ 0
$\Rightarrow (y+1)^2 - 4(y-1)^2 \ge 0 \Rightarrow -3y^2 + 10y - 3 \ge 0$

$\Rightarrow 3y^2 - 10y + 3 \le 0 \Rightarrow (3y-1)(y-3) \le 0 \Rightarrow \frac{1}{3} \le y \le 3$

$\therefore$ Range $= \left[\frac{1}{3}, 3\right]$

42. (b) : $5|x| - x^2 - 6 \ge 0 \Rightarrow x^2 - 5|x| + 6 \le 0$
when $x < 0$, $x^2 + 5x + 6 \le 0$, $-3 \le x \le -2$
when $x > 0$, $x^2 - 5x + 6 \le 0$, $2 \le x \le 3$
$x = 0$ will not satisfy the condition.
Domain is $[-3,-2] \cup [2,3]$.

43. (d) : $f(x) = 1 - \sin^2\frac{x}{4} + \sin\frac{x}{4}$

$= -\left\{\sin^2\frac{x}{4} - \sin\frac{x}{4}\right\} + 1 = -\left\{\left(\sin\frac{x}{4} - \frac{1}{2}\right)^2 - \frac{1}{4}\right\} + 1$

Maximum $f(x) = \frac{5}{4}$

Minimum $f(x) = \frac{5}{4} - \left(-1 - \frac{1}{2}\right)^2 = \frac{5}{4} - \frac{9}{4} = -1$

Range of $f(x) = \left[-1, \frac{5}{4}\right]$

44. (a) : Here, $f(x) = x^2 + 1 + \frac{1}{x^2+1} - 1$

$\because \quad x^2 + 1 + \frac{1}{x^2+1} \ge 2 \Rightarrow x^2 + \frac{1}{x^2+1} \ge 1$

$\therefore$ $f(x) \in [1, \infty)$

45. (b) : Let $y = f(x) = (5 - (x - 8)^5)^{1/3}$
Then $y^3 = 5 - (x - 8)^5 \Rightarrow (x - 8)^5 = 5 - y^3$
$\Rightarrow \; x = 8 + (5 - y^3)^{1/5}$
Let, $z = g(x) = 8 + 5(5 - x^3)^{1/5}$
Now, $f(g(x)) = (5 - [5 - x^3)^{1/5}]^5)^{1/3} = (5 - 5 + x^3)^{1/3} = x$
Similarly, we can show that $g(f(x)) = x$.
Hence, $g(x) = 8 + (5 - x^3)^{1/5}$ is the inverse of $f(x)$.

46. (b) : Let $t = x^3(x^3+3); t = (x^3+\frac{3}{2})^2 - \frac{9}{4} \in [-\frac{9}{4}, \infty)$
$f(x) = g(t) = t(t + 2) = (t + 1)^2 - 1$ is least when $t = -1$ and $-1 \in [-\frac{9}{4}, \infty)$ $\therefore$ min $f(x) = -1$

47. (b) : For function $f(x)$ to be defined we have $x^2 - 5x + 10 > 0$ ...(i) and $1 - \log_{10}(x^2 - 5x + 10) > 0$ ...(ii)
Now, (ii) $\Rightarrow \log_{10}(x^2 - 5x + 10) < 1 \Rightarrow x^2 - 5x + 10 < 10$
$\Rightarrow \; x^2 - 5x < 0 \; \Rightarrow x(x - 5) < 0 \Rightarrow 0 < x < 5$ ...(A)
Again, $x^2 - 5x + 10 > 0$ for all x, ...(B)
Since the discriminant of the corresponding equation $x^2 - 5x + 10 = 0$ is negative, so that the roots of the equation are imaginary.
Combining (A) and (B), we find that the domain of $f(x)$ is (0, 5).

48. (a) : Let $y_1 = \sin^{-1}\left[x^2+\frac{1}{2}\right]$ and $y_2 = \cos^{-1}\left[x^2-\frac{1}{2}\right]$
Then, $y = y_1 + y_2$.
Now, $y_1 = \sin^{-1}\left[x^2+\frac{1}{2}\right]$ is defined

if $-1 \le \left[x^2+\frac{1}{2}\right] \le 1 \Rightarrow -1 \le x^2+\frac{1}{2} < 2 \Rightarrow -\frac{3}{2} \le x^2 < \frac{3}{2}$ ...(i)

Again, $y_2 = \cos^{-1}\left[x^2-\frac{1}{2}\right]$ is defined

If $-1 \le \left[x^2-\frac{1}{2}\right] \le 1 \Rightarrow -1 \le x^2-\frac{1}{2} < 2 \Rightarrow -\frac{1}{2} \le x^2 < \frac{5}{2}$ ...(ii)

Taking the intersection of (i) and (ii), we find that
$-\frac{1}{2} \le x^2 < \frac{3}{2} \Rightarrow 0 \le x^2 < \frac{3}{2}$, since x^2 cannot be negative.

Now, for x^2 so that $\frac{1}{2} \le x^2+\frac{1}{2} \le 1$ and $-\frac{1}{2} \le x^2-\frac{1}{2} \le 0$
We have $Y = \sin^{-1}(0) + \cos^{-1}(-1) = 0 + \pi - \cos^{-1}(1) = \pi$
Similarly for $\frac{1}{2} \le x^2 < \frac{3}{2}$, we have
$y = \sin^{-1}(1) + \cos^{-1}(0) = \frac{\pi}{2}+\frac{\pi}{2} = \pi$.
Hence, the range of the given function is $[\pi]$.

49. (d) : It is given that $2^x + 2^y = 2 \; \forall \; x, y \in R$
Therefore, $2^x = 2 - 2^y < 2 \; \Rightarrow \; 0 < 2^x < 2$
Taking log for both side with base 2.
$\Rightarrow \; \log_2 0 < \log_2 2^x < \log_2 2$
Hence domain is $-\infty < x < 1$

50. (b) : $f(x + 1) = f(x + 5)$

51. (c) : 3 does not belong to the range of f implies 2 also cannot belong to range of f because, if $f(x) = 2$ for some $x \in R$. Then $f(x+p) = \frac{2-5}{2-3} = 3$ which is not in the range of f. Hence 2 and 3 are not in the range of f. If $f(x + 2p) = f(x)$, this implies $f(x) = f(x + p + p)$

$$= \frac{f(x+p)-5}{f(x+p)-3} = \frac{\frac{f(x)-5}{f(x)-3}-5}{\frac{f(x)-5}{f(x)-3}-3}$$

$$= \frac{-4f(x)+10}{-2f(x)+4} = \frac{2f(x)-5}{f(x)-2}$$

so that $[f(x) - 2]^2 = -1$ which is absurd. Therefore, $2p$ is not a period.

Again, $f(x+3p) = \frac{2f(x+p)-5}{f(x+p)-2} = \frac{3f(x)-5}{f(x)-1} \ne f(x)$

Now, $f(x + 4p) = f(x + 3p + p)$

$$= \frac{f(x+3p)-5}{f(x+3p)-3} = \frac{\frac{3f(x)-5}{f(x)-1}-5}{\frac{3f(x)-5}{f(x)-1}-3} = \frac{-2f(x)}{-2} = f(x)$$

Therefore, $4p$ is a period.

52. (a) : Given: $f(x) = (-1)^{[x]}$.
First of all, we sketch the graph of $f(x)$ with the help of piecewise defined functions as follows:

$$f(x) = (-1)^{[x]} = \begin{cases} 1; & -2 \le x < -1 \\ -1; & -1 \le x < 0 \\ 1; & 0 \le x < 1 \\ -1: & 1 \le x < 2 \\ 1; & 2 \le x < 3. \end{cases}$$

$\because$ The function $f(x)$ repeats its value after the least interval of 2.
Therefore, the function $f(x)$ is periodic with period 2.

53. (b) : $f(x)=x-\frac{1}{x}, \Rightarrow f(f(x))=\frac{x^4-3x^2+1}{x(x^2-1)}$

Since, we have $f(f(f(x)))=1 \Rightarrow f(f(x))=f^{-1}(1)=\frac{1+\sqrt{5}}{2}$

$\Rightarrow$ 2 values exist

or $f^{-1}(1)=\frac{1-\sqrt{5}}{2}$ $\Rightarrow$ 2 values exist

54. (c) : $\{x \in R : f^{-1}(x) = f(x)\} = \{x \in R : f f(x) = x\}$

$f(f(x)) = f(x)[f(x) - 1] = [x(x - 1)][x(x - 1) - 1]$

$= x(x - 1)[x^2 - x - 1]$

$f(f(x)) = x \Rightarrow x(x - 1)(x^2 - x - 1) = x$

$\Rightarrow x(x^3 - 2x^2) = 0 \Rightarrow x = 0, 2$

55. (c) : We have, $f(x)=2\sin\left(x-\frac{\pi}{6}\right)+2$

$\because$ f is one–one and onto $\therefore$ f is invertible

Now, $fof^{-1}(x) = x \Rightarrow 2\sin\left(f^{-1}(x)-\frac{\pi}{6}\right)+2=x$

$f^{-1}(x)=\sin^{-1}\left(\frac{x}{2}-1\right)+\frac{\pi}{6}$ $\left(\because \left|\frac{x}{2}-1\right| \le 1 \forall x \in [0,4]\right)$

Also, $\sin^{-1}\alpha + \cos^{-1}\alpha = \frac{\pi}{2}$

$\Rightarrow f^{-1}(x)=\frac{\pi}{2}-\cos^{-1}\frac{x-2}{2}+\frac{\pi}{6}=\frac{2\pi}{3}-\cos^{-1}\left(\frac{x-2}{2}\right)$

56. (b) : Let $y=f(x)=1+\alpha x \Rightarrow x=\frac{y-1}{\alpha}$

$\Rightarrow f^{-1}(x)=\frac{x-1}{\alpha}$

Now, $f(x)=f^{-1}(x) \Rightarrow \frac{x-1}{\alpha}=1+\alpha x \Rightarrow x-1=\alpha+\alpha^2 x$

Equating the coefficient of x, we get

$\alpha^2 = 1$ and $\alpha = -1 \Rightarrow \alpha = -1$

57. (c) : We have, $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$

$\Rightarrow 4f(x^2)+6f\left(\frac{1}{x^2}\right)=2x^2-2$ ...(i)

Also, $2f\left(\frac{1}{x^2}\right)+3f(x^2)=\frac{1}{x^2}-1$

$\Rightarrow 9f(x^2)+6f\left(\frac{1}{x^2}\right)=\frac{3}{x^2}-3$ ...(ii)

$\text{(ii)}-\text{(i)} \Rightarrow 5f(x^2)=\frac{3}{x^2}-2x^2-1 \Rightarrow f(x^2)=\frac{3-2x^4-x^2}{5x^2}$

58. (a) : We have, $g(x)\, g(y) = g(x)+ g(y)+g(xy)-2$ ...(i)

Putting $x = 1, y = 2$ in (i), we have

$g(1)\, g(2) = g(1) + g(2) + g(2) - 2$

$\Rightarrow 5g(1) = 8 + g(1) \therefore g(1) = 2$

Also, replacing y by $\frac{1}{x}$ in (i), we get

$g(x)g\left(\frac{1}{x}\right)=g(x)+g\left(\frac{1}{x}\right)+g(1)-2$

$\Rightarrow g(x)g\left(\frac{1}{x}\right)=g(x)+g\left(\frac{1}{x}\right)$

$\Rightarrow g(x) = 1 \pm x^n$ ...(ii)

Put $x = 2$ in (ii), we get $\pm 2^n = 2^2$

Taking +ve sign, we set $n = 2$

$\therefore g(x) = 1 + x^2 \Rightarrow g(3) = 1 + 3^2 = 10$

59. (b)

60. (c) : Taking $x = y = 1$, we get

$f(1)f(1) - f(1) = 2 \Rightarrow f^2(1) - f(1) - 2 = 0$

$\Rightarrow (f(1) - 2)(f(1) + 1) = 0 \Rightarrow f(1) = 2$ (as $f(1) > 0$)

Taking $y = 1$, we get

$f(x).f(1) - f(x) = x + 1$

$\Rightarrow f(x) = x + 1 \Rightarrow f^{-1}(x) = x - 1$

$\therefore f(x).f^{-1}(x) = x^2 - 1$

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. If p^{th}, q^{th} and r^{th} terms of an H.P., be respectively x, y and z, then prove that $(p - q)xy + (q - r)yz + (r - p)zx = 0$. *(Ankit Jain, Madhya Pradesh)*

Ans. Since, x, y, z are respectively the p^{th}, q^{th} and r^{th} terms of an H.P., hence, $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ will respectively be the p^{th}, q^{th} and r^{th} terms of an A.P.
If a and d be the first term and common difference of the corresponding A.P., then we have

$$\frac{1}{x} = a + (p-1)d \qquad \text{...(i)}$$

$$\frac{1}{y} = a + (q-1)d \qquad \text{...(ii)}$$

and $$\frac{1}{z} = a + (r-1)d \qquad \text{...(iii)}$$

Subtracting equation (ii) from equation (i), we have

$$\frac{1}{x} - \frac{1}{y} = (p-q)d$$

i.e., $$xy(p-q) = \frac{y-x}{d} \qquad \text{...(iv)}$$

Similarly, subtracting equation (iii) from equation (ii), we have,

$$xz(q-r) = \frac{z-y}{d} \qquad \text{...(v)}$$

and subtracting equation (i) from equation (iii), we have

$$zx(r-p) = \frac{x-z}{d} \qquad \text{...(vi)}$$

Adding (iv), (v) and (vi), we get the desired result.

2. If z_1, z_2 are two complex numbers and ω^k, $k = 0, 1, ..., n-1$ are the n^{th} roots of unity, then show that

$$\sum_{k=0}^{n-1} |z_1 + z_2\omega^k|^2 = n(|z_1|^2 + |z_2|^2)$$

(Pratiksha John, Kerala)

Ans. We have, $|z_1 + |z_2\omega^k|^2 = (z_1 + z_2\omega^k)\overline{(z_1 + z_2\omega^k)}$

$= (z_1 + z_2\omega^k)(\bar{z}_1 + \bar{z}_2\omega^{-k})$

$\left[\because \omega^k = e^{i(2\pi k/n)} \Rightarrow \overline{(\omega^k)} = e^{-i(2\pi k/n)} = \omega^{-k}\right]$

$= |z_1|^2 + |z_2|^2 + \bar{z}_1 z_2\omega^k + z_1\bar{z}_2\omega^{-k}$

Therefore, we have

$$\sum_{k=0}^{n-1} |z_1 + z_2\omega^k|^2 = n(|z_1|^2 + |z_2|^2) + \bar{z}_1 z_2 \sum_{k=0}^{n-1} \omega^k + z_1\bar{z}_2 \sum_{k=0}^{n-1} \omega^{-k}$$

$$= n(|z_1|^2 + |z_2|^2). \qquad \left[\because \sum_{k=0}^{n-1} \omega^k = 0 = \sum_{k=0}^{n-1} \omega^{-k}\right]$$

3. Show that the integral part of $(5\sqrt{5} + 11)^{67}$ is even. *(Indu Mishra, Delhi)*

Ans. Let $(5\sqrt{5} + 11)^{67} = I + f$ where I and f are the integral and the fractional parts of $(5\sqrt{5} - 11)^{67}$ respectively.

Let $(5\sqrt{5} - 11)^{67} = g$ where g is a fraction. Since $0 < 5\sqrt{5} - 11 < 1$, therefore $0 < (5\sqrt{5} - 11)^{67} < 1$ for every positive integer n.

We have, $I + f - g = (5\sqrt{5} + 11)^{67} - (5\sqrt{5} - 11)^{67}$

$= 2[C_1(5\sqrt{5})^{66} \cdot 11 + C_3(5\sqrt{5})^{64} \cdot 11^3 + ... + C_{67} \cdot 11^{67}]$

[where $C_r = {}^{67}C_r$]

i.e., $I + f - g = 2k, k \in Z^+$ ...(i)

i.e., $f - g = 2k - I =$ an integer ...(ii)

Since $0 < f, g < 1$, therefore we have

$-1 < f - g < 1$ ...(iii)

Thus, using results (ii) and (iii), we have

$f - g = 0$

[since the only integral value in $(-1, 1)$ is 0] ...(iv)

Putting result (iv) in equation (i) we have

$I = 2k - (f - g) = 2k =$ an even integer.

Solution Sender of Maths Musing

SET-181

- Santanu Singh (West Bengal)

CONCEPT MAP

SEQUENCES AND SERIES

Class XI

Sequence
A sequence is a function from natural number N (domain) to real numbers (codomain)

Series
If $a_1, a_2, ..., a_n$ is a sequence, then the expansion $a_1 + a_2 + ... + a_n$ is called the series.

Progression
If the terms of a sequence are written under specific conditions, then the sequence is called progression.

Types of Progression

Sum of n terms of Special Series

- Sum of n natural numbers $\sum n = \frac{n(n+1)}{2}$
- Sum of squares of n natural numbers $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of n natural numbers $\sum n^3 = \frac{n^2(n+1)^2}{4} = \left(\sum n\right)^2$
- $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$
- $1\cdot 2 + 2\cdot 3 + ... + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Arithmetic Progression (A.P.)

A sequence whose terms increases or decreases by a fixed number.

n^{th} term : $T_n = a + (n-1)d$ where d (common difference) $= T_n - T_{n-1}$, a = first term,

n^{th} term from end

$T_n' = l - (n-1)d$, where l = last term

Sum of n terms

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[a + l]$

Note
- 3 terms in an A.P. $a - d, a, a + d, a + 2d$
- 4 terms in an A.P. $a - 2d, a - d, a, a + d, a + 2d$

Basic Properties
- If $a_1, a_2, a_3, ..., a_n$ are in A.P. with common difference d, then
  - $a_1 \pm x, a_2 \pm x, ..., a_n \pm x$ are also in A.P. where x is constant.
  - $ka_1, ka_2, ..., ka_n$ are also in A.P. with common difference kd.
  - $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, ..., \frac{a_n}{k}$ are also in A.P. with common difference, $\frac{d}{k}, k \neq 0$.

Geometric Progression (G.P.)

A sequence of non-zero numbers for which the ratio of a term to its just preceding term is always constant.

n^{th} term : $T_n = ar^{n-1}$, where r(common ratio) $= T_n/T_{n-1}$, a = first term

n^{th} term from end : $T_n' = l/r^{n-1}$, l = last term

Sum of n terms

$$S_n = \begin{cases} \frac{a(r^n - 1)}{r-1}, r > 1 \\ \frac{a(1-r^n)}{1-r}, r < 1 \\ an, \quad r = 1 \end{cases} ; S_\infty = \frac{a}{1-r}, |r| < 1$$

Note
- 3 terms in G.P. $a/r, a, ar$.
- 4 terms in G.P. $a/r^2, a/r, a, ar$.

Basic Properties
- If $a_1, a_2, a_3, ..., a_n$ are in G.P., with common ratio r then
  - $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ..., \frac{1}{a_n}$ also in G.P., with common ratio $1/r$.
  - $ka_1, ka_2, ka_3, ..., ka_n$ or $\frac{a_1}{k}, \frac{a_2}{k}, \frac{a_3}{k}, ...$ $(k \neq 0)$ also in G.P., with common ratio r.
  - $a_1^n, a_2^n, a_3^n, ..., a_n^n$ also in G.P. with common ratio r^n.
  - $a_1 a_n = a_k \cdot a_{n-k+1}$ $\forall k = 1, 2, 3, ..., n-1$

Means

Arithmetic mean (A.M.)
- For two numbers a and b, A.M. is $\frac{a+b}{2}$.
- $A_r = a + r\left(\frac{b-a}{n+1}\right), \forall r = 1, 2, ..., n$

  where $A_1, ..., A_n$ are n arithmetic means inserted between two numbers a and b.

Geometric mean (G.M.)
- For two numbers a and b, G.M. is $\sqrt{ab}$.
- $G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}} \forall k = 1, 2, 3, ..., n$

  where $G_1, ... , G_n$ are n geometric means inserted between two numbers a and b.

CONCEPT MAP

DEFINITE INTEGRALS AND APPLICATION OF INTEGRALS

Class XII

DEFINITE INTEGRALS

For any two values a and b, we have $\int_a^b f(x)\,dx = [F(x) + c]_a^b = F(b) - F(a)$

Limit of Sum

$\int_a^b f(x)\,dx = \lim_{h\to 0} h[f(a) + f(a+h) + ... + f(a + (n-1)h)]$,

where $h = \frac{b-a}{n} \to 0$ as $n \to \infty$

Fundamental Theorem of Calculus

- **First Fundamental Theorem :** Let $f(x)$ be a continuous function in the closed interval $[a, b]$ and let $A(x)$ be the area function. Then $A'(x) = f(x)$, for all $x \in [a, b]$.
- **Second Fundamental Theorem :** Let $f(x)$ be a continuous function in the closed interval $[a, b]$ and $F(x)$ be an integral of $f(x)$, then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

Solving by Substitution

When definite integral is to be found by substitution, change the lower and upper limits of integration. If substitution is $t = f(x)$ and lower limit of integration is a and upper limit is b, then new lower and upper limits will be $f(a)$ and $f(b)$ respectively.

Properties

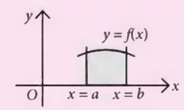
- $\int_a^b f(x)dx = \int_a^b f(t)dt$
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$. In particular $\int_a^a f(x)dx = 0$
- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where $a < c < b$
- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ • $\int_0^a f(x)dx = \int_0^a f(a-x)dx$
- $\int_{-a}^a f(x)dx = \begin{cases} 0 & , \text{if } f(-x) = -f(x) \\ 2\int_0^a f(x)dx & , \text{if } f(-x) = f(x) \end{cases}$
- $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$
- $\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a-x) = f(x) \\ 0, & \text{if } f(2a-x) = -f(x) \end{cases}$

APPLICATION OF INTEGRALS

Area Under Simple Curves

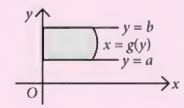
- Area $= \int_a^b y\,dx$

  $= \int_a^b f(x)dx$ (where $b > a$)

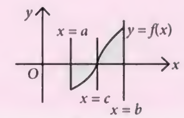


- Area $= \int_a^b x\,dy$

  $= \int_a^b g(y)dy$ (where $b > a$)



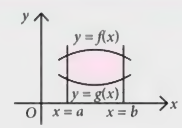
- Area $= \left|\int_a^c f(x)dx\right| + \int_c^b f(x)dx$



Area Between Two Curves

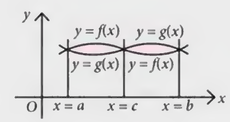
- Area $= \int_a^b [f(x) - g(x)]dx$,

  $f(x) \geq g(x)$ in $[a, b]$



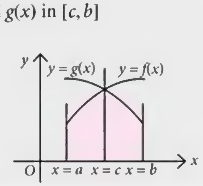
- Area $= \int_a^c [f(x) - g(x)]dx$

  $+ \int_c^b [g(x) - f(x)]dx$



where $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$

- Area $= \int_a^c f(x)dx + \int_c^b g(x)dx$



Exam Dates
OFFLINE : 8th April
ONLINE : 15th & 16th April

JEE Main 2018
MOCK TEST PAPER

1. A letter is taken out at random from 'ASSISTANT' and another letter taken out from the letter of the word 'STATISTICS'. The probability that they are identical letters, is
(a) $\frac{13}{90}$ (b) $\frac{1}{45}$
(c) $\frac{19}{90}$ (d) none of these

2. The number of ways of arranging the word 'ARRANGE' so that neither 2A's nor 2R's occurs together are
(a) 900 (b) 240 (c) 660 (d) 7!

3. A man has seven relatives 4 of them are ladies and 3 gentlemen, his wife has 7 relatives out of which 3 are ladies and 4 gentlemen. The number of ways in which they can invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of the man's relative and 3 of the wife's relative are
(a) 144 (b) 340 (c) 484 (d) 485

4. The value of $\tan i\left[\log_e\left(\frac{a-ib}{a+ib}\right)\right]=$
(a) 0 (b) 1
(c) -1 (d) none of these

5. Let $A=\begin{bmatrix}0 & 2b & c\\ a & b & -c\\ a & -b & c\end{bmatrix}$ be an orthogonal matrix the values of a, b, c are related with
(a) $a=\pm\frac{1}{\sqrt{2}},b=\pm\frac{1}{\sqrt{6}},c=\pm\frac{1}{\sqrt{3}}$
(b) $a=\pm\frac{1}{\sqrt{2}},c=\pm\frac{1}{\sqrt{6}}$
(c) $a=\pm\frac{1}{\sqrt{2}},b=\pm\frac{1}{\sqrt{6}}$ (d) none of these

6. If maximum and minimum values of the determinant $\begin{vmatrix}1+\cos^2 x & \sin^2 x & \cos 2x\\ \cos^2 x & 1+\sin^2 x & \cos 2x\\ \cos^2 x & \sin^2 x & 1+\cos 2x\end{vmatrix}$ are α and β respectively then
(a) $\alpha^2+\beta^{101}=10$ (b) $\alpha^3-\beta^{99}=26$
(c) $2\alpha^2-18\beta^{11}=0$ (d) all of these

7. If $\int\frac{dx}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}$ $=A\tan^{-1}(B\tan x+C)$ then
(a) $A=\frac{1}{4},B=\frac{1}{2},C=1$ (b) $A=\frac{1}{2},B=\frac{1}{4},C=1$
(c) $A=1,B=\frac{1}{2},C=\frac{1}{4}$ (d) $A=\frac{1}{4},B=1,C=\frac{1}{2}$

8. If $I(x)=\int_0^{\pi/2}\frac{dx}{\sqrt{1+\sin^3 x}}$ and $a\le I(x)\le b$, then
(a) $a=\pi\sqrt{2},b=\frac{\pi}{2}$ (b) $a=\frac{\pi}{2\sqrt{2}},b=\frac{\pi}{4}$
(c) $a=\frac{\pi}{2\sqrt{2}},b=\frac{\pi}{2}$ (d) $a=\frac{\pi}{\sqrt{2}},b=\pi$

9. The area of the parallelogram whose sides are along the straight lines $y=3x+5$, $y=3x+2$, $y=5x+4$ and $y=5x-1$ equals
(a) $\frac{15}{2}$ sq. units (b) 15 sq. units
(c) $\frac{15}{\sqrt{10}\sqrt{26}}$ sq. units (d) none of these

By : Sankar Ghosh, S.G.M.C, Mob : 09831244397.

10. Tangent drawn from the point (4, 4) to the circle $x^2 + y^2 - 2x - 2y - 7 = 0$ meet the circle at A and B. The length of the chord AB equals
(a) $4\sqrt{3}$ (b) $2\sqrt{3}$ (c) $2\sqrt{6}$ (d) $3\sqrt{2}$

11. The set of values of m for which a chord of slope m of the circle $x^2 + y^2 = 16$ touches the parabola $y^2 = 8x$
(a) $(-\infty, -1) \cup (1, \infty)$
(b) $(-\infty, \infty)$
(c) $\left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$
(d) none of these

12. If the normal at an end of a latus rectum passes through the extremity of the minor axis then eccentricity of the ellipse is given by
(a) $e^2 + e + 1 = 0$ (b) $e^4 + e^2 + 1 = 0$
(c) $e^4 - e^2 - 1 = 0$ (d) $e^4 + e^2 - 1 = 0$

13. If the $(m + 1)^{th}$, $(n + 1)^{th}$, and $(r + 1)^{th}$ term of an A.P. are in G.P. and m, n, r are in H.P., then the ratio of the first term of the A.P. to its common difference is
(a) $n/2$ (b) $-n/2$ (c) $n/4$ (d) $-2n$

14. If $\log_{\sqrt{7}} x + \log_{\sqrt[3]{7}} x + \log_{\sqrt[4]{7}} x + \ldots$ upto 20^{th} term $= 230$ then the value of x equals
(a) 7^2 (b) 7
(c) 7^3 (d) none of these

15. If α, β are roots of the equation $p(x^2 - x) + x + 5 = 0$ and p_1, p_2 are two values of p for which the roots α, β are connected by the relation $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$ then the value of $\frac{p_1}{p_2} + \frac{p_2}{p_1} =$
(a) 254 (b) 0 (c) 245 (d) -254

16. A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If the graph passes through the point (2, 1) and at that point the tangent to the curve is $y = 3x - 5$, then the function is
(a) $(x + 1)^3$ (b) $(x - 1)^3$
(c) $(x - 1)^2$ (d) $(x + 1)^2$

17. The function $f(x) = \begin{cases} \frac{\sin^3 x^2}{x} & \forall x \neq 0 \\ 0 & \forall x = 0 \end{cases}$
(a) is continuous but not derivable at $x = 0$
(b) neither continuous nor derivable at $x = 0$
(c) continuous and differentiable at $x = 0$
(d) none of these.

18. If $f(x) = \int_{x^2}^{x^3} \frac{dt}{\log t}, x > 0$ then
(a) $f(x)$ is maximum at $x = 1$
(b) $f(x)$ is an increasing function $\forall\ x \in R^+$ only
(c) $f(x)$ is minimum at $x = 1$
(d) $f(x)$ is neither maximum nor minimum at $x = 1$ but an increasing function for all x belong to R

19. If A_1 and A_2 respectively represents the area bounded by the curves $f(x, y) : 4x^2 \leq y \leq 3x$ and $g(x, y) : 4x^2 \leq y \leq |3x|$ then $A_1 : A_2$ equals
(a) 2 : 1 (b) 3 : 1 (c) 1 : 2 (d) 1 : 3

20. Solution of differential equation $\frac{dt}{dx} = \frac{t\left(\frac{dg(x)}{dx}\right) - t^2}{g(x)}$ is
(a) $t = \frac{g(x)+c}{x}$ (b) $t = \frac{g(x)}{x} + c$
(c) $t = \frac{g(x)}{x+c}$ (d) $t = g(x) + x + c$

21. Let Y be a set of all complex numbers such that $|z| = 1$ and defined relation R on Y by $z_1 R z_2$ is $|\arg z_1 - \arg z_2| = \frac{2\pi}{3}$ then R is
(a) symmetric (b) transitive
(c) anti-symmetric (d) reflexive

22. If $\lim_{x \to 0} \frac{f(x)}{x} = 2$ where $f(x) = \min.\{\sin\sqrt{[m]}x, |x|\}$ and $[\cdot]$ greatest integer then
(a) $m \in \{4\}$ (b) $m \in [4, 5]$
(c) $m \in [4, 5)$ (d) $m = \{5\}$

23. The value of $3\tan^6 10° - 27\tan^4 10° + 33\tan^2 10°$ is
(a) 0 (b) -1
(c) 1 (d) none of these

24. The angle between the line $\vec{r} = (2\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} - 3\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (3\hat{i} - 2\hat{j} + 5\hat{k}) = 4$
(a) $\sin^{-1}\left(\frac{11}{\sqrt{323}}\right)$ (b) $\sin^{-1}\left(\frac{22}{\sqrt{323}}\right)$
(c) $\sin^{-1}\left(\frac{22}{\sqrt{646}}\right)$ (d) none of these

25. The line $\frac{x-3}{2}=\frac{y-4}{5}=\frac{z-6}{7}$
(a) lies in $3x + 2y + 4z - 6 = 0$
(b) is parallel to $2x - 5y + 3z = 9$
(c) is perpendicular to $2x - 5y + 3z = 0$
(d) passing through (1, 2, 3)

26. Let $\vec{a}=\alpha_1\hat{i}+\alpha_2\hat{j}+\alpha_3\hat{k}$, $\vec{b}=\beta_1\hat{i}+\beta_2\hat{j}+\beta_3\hat{k}$ and $\vec{c}=\gamma_1\hat{i}+\gamma_2\hat{j}+\gamma_3\hat{k}$, $|\vec{a}|=2\sqrt{2}$, $\vec{a}$ makes angle $\pi/3$ with plane of $\vec{b}$ and $\vec{c}$ and angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, then $\begin{vmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{vmatrix}^n$ is equal to (n is even natural number)
(a) $\left(\frac{|\vec{b}||\vec{a}|}{6}\right)^{n/2}$ (b) $\left(\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}}\right)^n$
(c) $\frac{\left(|\vec{b}||\vec{c}|\right)^{n/2}}{\sqrt{32^n}}$ (d) none of these

27. The mean deviation and S.D. about actual mean of the series $a, a + d, a + 2d, ..., a + 2nd$ are respectively
(a) $\frac{n(n+1)d}{2n+1}, \sqrt{\frac{n(n-1)}{3}}\cdot d$
(b) $\frac{n(n-1)}{3}, \frac{n(n+1)}{2n}\cdot d$
(c) $\frac{n(n+1)d}{2n+1}, \sqrt{\frac{n(n+1)}{3}}\cdot d$
(d) $\frac{n(n-1)d}{2n-1}, \sqrt{\frac{n(n-1)}{3}}\cdot d$

28. Given $\pi<\theta<\frac{3\pi}{2}$ then the value of expression $\sqrt{4\sin^4\theta+\sin^2 2\theta+4\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}$ is
(a) 2 (b) $2 + 4\sin\theta$
(c) $2 - 4\sin\theta$ (d) none of these

29. In a ΔABC, $\angle B = 90°$ and $b + a = 4$. The area of the triangle is the maximum when $\angle C$ is
(a) $\pi/4$ (b) $\pi/6$
(c) $\pi/3$ (d) none of these

30. If there is an error in k% in measuring the edge of a cube then the percent error in estimating its volume is
(a) k (b) $3k$
(c) $k/3$ (d) none of these

SOLUTIONS

1. **(c)** : In the word 'ASSISTANT' there are SSS AA I TT N and in 'STATISTICS" there are A C II SSS TTT. The non common letters are N and C. The identical letters are A, I, S, T.

Probability of getting $A=\frac{^2C_1}{^9C_1}\times\frac{^1C_1}{^{10}C_1}=\frac{1}{45}$

Probability of getting $I=\frac{^1C_1}{^9C_1}\times\frac{^2C_1}{^{10}C_1}=\frac{1}{45}$

Probability of getting $S=\frac{^3C_1}{^9C_1}\times\frac{^3C_1}{^{10}C_1}=\frac{1}{10}$

Probability of getting $T=\frac{^2C_1}{^9C_1}\times\frac{^3C_1}{^{10}C_1}=\frac{1}{15}$

$\therefore$ Required probability $=\frac{1}{45}+\frac{1}{45}+\frac{1}{10}+\frac{1}{15}=\frac{19}{90}$

2. **(c)** : The number of ways in which 2R's are never together $=\frac{7!}{2!2!}-\frac{6!}{2!}=\frac{5040}{4}-\frac{720}{2}=1260-360=900$

The number of ways in which 2A's are together but not two 2R's $=4!\times\frac{^5P_2}{2}=24\times10=240$

Now, there are in all 900 arrangements in each of which the two R's never together. In any one such arrangement, either the two A's are together or two A's are not together.
Since number of all such arrangements in which two A's are together = 240, therefore, the number of arrangements in which neither two R's nor 2A's are together = 900 – 240 = 660

3. **(d)** : The number of ways of selections of 3 man's relative and 3 wife's relative are shown in the following table

Case	Man's relative(7)		Wife's relative(7)	
	3 men	4 women	4 men	3 women
(i)	0	3	3	0
(ii)	1	2	2	1
(iii)	2	1	1	2
(iv)	3	0	0	3

The required number of ways are
$(^3C_0 \times {}^4C_3 \times {}^4C_3 \times {}^3C_0) + (^3C_1 \times {}^4C_2 \times {}^4C_2 \times {}^3C_1)$
$+ (^3C_2 \times {}^4C_1 \times {}^4C_1 \times {}^3C_2) + (^3C_3 \times {}^4C_0 \times {}^4C_0 \times {}^3C_3)$
$= 16 + 324 + 144 + 1 = 485$

4. (d) : $\tan i\left[\log_e\left(\frac{a-ib}{a+ib}\right)\right]$

$= \tan i\left[\log_e\left(\frac{\cos\theta - i\sin\theta}{\cos\theta + i\sin\theta}\right)\right]$

(putting $a = r\cos\theta$ and $b = r\sin\theta$)

$= \tan[i\log(\cos 2\theta - i\sin 2\theta)] = \tan(i\log_e e^{-2i\theta})$

$= \tan(-i^2 2\theta) = \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2ab}{a^2-b^2}$

5. (a) : We have,

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix} \therefore A^t = \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$$

As A is an orthogonal matrix, therefore $AA^t = I$

$$\Rightarrow \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}\begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4b^2+c^2 & 2b^2-c^2 & -2b^2+c^2 \\ 2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ -2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, from the definition of equality of two matrices we have, $4b^2 + c^2 = 1$, $2b^2 - c^2 = 0$, $a^2 + b^2 + c^2 = 1$

On solving the above equations we get,

$$a = \pm\frac{1}{\sqrt{2}}, b = \pm\frac{1}{\sqrt{6}}, c = \pm\frac{1}{\sqrt{3}}$$

6. (d) : Let $d = \begin{vmatrix} 1+\cos^2 x & \sin^2 x & \cos 2x \\ \cos^2 x & 1+\sin^2 x & \cos 2x \\ \cos^2 x & \sin^2 x & 1+\cos 2x \end{vmatrix}$

Applying $R_1 \leftrightarrow R_1 - R_2$ and $R_2 \leftrightarrow R_2 - R_3$, we get

$d = 1(1+\cos 2x + \sin^2 x) + 1(\cos^2 x) + 0$

$= 1 + \cos 2x + \sin^2 x + \cos^2 x = 2 + \cos 2x$

Now, we know that $-1 \le \cos 2x \le 1$

$\Rightarrow 2-1 \le 2+\cos 2x \le 2+1 \Rightarrow 1 \le 2+\cos 2x \le 3$

$\Rightarrow \beta \le 2+\cos 2x \le \alpha \quad \therefore \quad \alpha = 3, \beta = 1$

Now, $\alpha^2 + \beta^{101} = 9 + 1 = 10$

$\alpha^3 - \beta^{99} = 27 - 1 = 26$

$2\alpha^2 - 18\beta^{11} = 18 - 18 = 0$

7. (d) : Here $\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x}$

$= \int \frac{\sec^2 x\, dx}{4\tan^2 x + 4\tan x + 5} = \int \frac{dz}{(2z+1)^2 + 2^2}$, where $z = \tan x$

$= \frac{1}{2}\cdot\frac{1}{2}\tan^{-1}\left(\frac{2z+1}{2}\right) = \frac{1}{4}\tan^{-1}\left(\tan x + \frac{1}{2}\right)$

$= A\tan^{-1}(B\tan x + C)$

$\therefore A = \frac{1}{4}, B = 1, C = \frac{1}{2}$

8. (c) : We have, $0 \le x \le \frac{\pi}{2}$

$\Rightarrow \sin 0 \le \sin x \le \sin\frac{\pi}{2} \Rightarrow 0 \le \sin x \le 1$

$\Rightarrow 0 \le \sin^3 x \le 1$

$\Rightarrow 1 \le 1 + \sin^3 x \le 2$

$\Rightarrow \frac{1}{2} \le \frac{1}{1+\sin^3 x} \le 1$

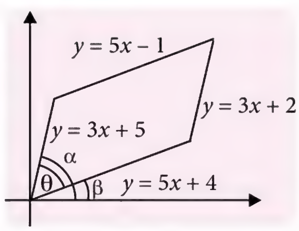
$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}}\, dx \le \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1+\sin^3 x}} \le \int_0^{\frac{\pi}{2}} 1 dx$

$\Rightarrow \frac{\pi}{2\sqrt{2}} \le I(x) \le \frac{\pi}{2} \Rightarrow a = \frac{\pi}{2\sqrt{2}}, b = \frac{\pi}{2}$

9. (a) : Area of the parallelogram is $P_1P_2\operatorname{cosec}\theta$, where P_1, P_2 are perpendicular distances between the two pair of parallel sides and θ be the angle between two adjacent sides.

Let the distance between the parallel lines $y = 5x + 4$ and $y = 5x - 1$ be P_1

$\therefore \quad P_1 = \frac{4-(-1)}{\sqrt{1^2+5^2}} = \frac{5}{\sqrt{26}}$



Also, let distance between the parallel lines $y = 3x + 5$ and $y = 3x + 2$ be P_2

$\therefore \quad P_2 = \frac{5-2}{\sqrt{1^2+3^2}} = \frac{3}{\sqrt{10}}$

Again, $\theta = \alpha - \beta$, $\tan\alpha = \frac{3}{1}$, $\tan\beta = \frac{5}{1}$

$\therefore \quad \sin\theta = \sin(\alpha - \beta) = \cos\alpha\cos\beta(\tan\alpha - \tan\beta)$

$= (3-5)\times\frac{1}{\sqrt{10}}\cdot\frac{1}{\sqrt{26}} \quad \left[\because \cos\alpha = \frac{1}{\sqrt{10}}, \cos\beta = \frac{1}{\sqrt{26}}\right]$

Therefore, $\operatorname{cosec}\theta = \dfrac{\sqrt{10}\sqrt{26}}{2}$

(neglecting the negative sign).

$\therefore$ Required area $= P_1P_2 \operatorname{cosec}\theta$

$$= \frac{5}{\sqrt{26}} \times \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}\sqrt{26}}{2} = \frac{15}{2} \text{sq.units.}$$

10. (d) : The equation of circle is

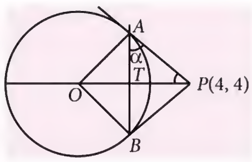
$x^2 + y^2 - 2x - 2y - 7 = 0 \Rightarrow (x-1)^2 + (y-1)^2 = (3)^2.$

$\therefore$ Centre (1, 1) and radius = 3

Now, $PA = PB = \sqrt{16+16-8-8-7} = \sqrt{9} = 3$

Again, $OP = (OA)^2 + (AP)^2$

$= \sqrt{9+9} = 3\sqrt{2}$



Now, in ΔATP, $\dfrac{AT}{AP} = \cos\alpha$

$\Rightarrow \quad AT = AP\cos\alpha = 3\cos 45°$

$\therefore \quad AB = 2AT = 2 \times 3 \times \dfrac{1}{\sqrt{2}} = 3\sqrt{2}$

11. (c) : For parabola $y^2 = 4ax$, the line $y = mx + c$ will be tangent if $c = \dfrac{a}{m}$.

$\therefore$ Equation of tangent to the parabola $y^2 = 8x$ is

$$y = mx + \frac{2}{m}$$

Now line to be chord of the circle $x^2 + y^2 = 16$ if distance from (0, 0) to the line will be less than the radius of the circle.

$$\therefore \quad \frac{\frac{2}{m}}{\sqrt{1+m^2}} < 4 \Rightarrow \frac{2}{m} < 4\sqrt{1+m^2}$$

$$\Rightarrow 1 < 4m^2(1+m^2) \Rightarrow 4m^4 + 4m^2 - 1 > 0$$

$$\Rightarrow m^4 + m^2 - \frac{1}{4} > 0 \Rightarrow \left(m^2 + \frac{1}{2}\right)^2 - \frac{1}{2} > 0$$

$$\left(\left(m^2 + \frac{1}{2}\right) + \frac{1}{\sqrt{2}}\right)\left(\left(m^2 + \frac{1}{2}\right) - \frac{1}{\sqrt{2}}\right) > 0$$

$$m^2 < -\frac{1}{2} + \frac{\sqrt{2}}{2} \text{ or } m^2 > \frac{\sqrt{2}-1}{2}$$

$$\Rightarrow \quad m \in \left(-\infty, -\sqrt{\frac{\sqrt{2}-1}{2}}\right) \cup \left(\sqrt{\frac{\sqrt{2}-1}{2}}, \infty\right)$$

12. (d) : Let equation of ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

and $P\left(ae, \dfrac{b^2}{a}\right)$ be one end of a latus rectum through $S(ae, 0)$.

Now, equation of normal L is $\dfrac{a^2x}{ae} - \dfrac{b^2y}{\frac{b^2}{a}} = a^2 - b^2$

$$\Rightarrow \quad \frac{ax}{e} - ay = a^2e^2 \quad \Rightarrow \quad \frac{x}{e} - y = ae^2$$

This normal is passing through $L'(0, -b)$

$\therefore \quad b = ae^2 \Rightarrow b^2 = a^2e^4$

$\therefore \quad a^2(1 - e^2) = a^2e^4 \quad \Rightarrow \quad e^4 + e^2 - 1 = 0$

13. (b) : Let the first term and common difference be α and β respectively.

$\therefore \quad t_{m+1} = \alpha + m\beta, \quad t_{n+1} = \alpha + n\beta, \quad t_{r+1} = \alpha + r\beta$

Again, we have t_{m+1}, t_{n+1} and t_{r+1} are in G.P.

$\therefore \; (\alpha + n\beta)^2 = (\alpha + m\beta)(\alpha + r\beta)$

$$\Rightarrow \quad \alpha\beta(2n - m - r) = \beta^2(mr - n^2) \Rightarrow \frac{\alpha}{\beta} = \frac{(mr - n^2)}{2n - m - r}$$

$$\left[\because m, n, r \text{ are in H.P. } \therefore \frac{2}{n} = \frac{1}{m} + \frac{1}{r} \Rightarrow 2mr = n(m+r)\right]$$

$$\Rightarrow \quad \frac{\alpha}{\beta} = \frac{2mr - 2n^2}{2(2n - m - r)} = \frac{mn + nr - 2n^2}{2(2n - m - r)}$$

$$= \frac{n(m + r - 2n)}{-2(m + r - 2n)} = -\frac{n}{2}$$

14. (b) : $\log_{\sqrt{7}} x + \log_{\sqrt[3]{7}} x + \log_{\sqrt[4]{7}} x +$

... upto 20 terms = 230

$\Rightarrow \; \log_7 x\{(1 + 2 + 3 + 4 + ... + 20 + 21) - 1\} = 230$

$\Rightarrow \; 230(\log_7 x) = 230 \Rightarrow \log_7 x = 1 \Rightarrow x = 7$

15. (a) : Here the given equation is

$p(x^2 - x) + x + 5 = 0 \quad \Rightarrow \quad px^2 - (p - 1)x + 5 = 0$

$$\therefore \; \alpha + \beta = \frac{p-1}{p} \text{ and } \alpha\beta = \frac{5}{p}$$

$$\text{Now } \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5} \Rightarrow \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5} \quad \Rightarrow \quad \frac{(p-1)^2 - 10p}{5p} = \frac{4}{5}$$

$$\Rightarrow \quad p^2 - 16p + 1 = 0 \quad \therefore \quad p_1 + p_2 = 16 \text{ and } p_1p_2 = 1$$

$$\text{Now, } \frac{p_1}{p_2} + \frac{p_2}{p_1} = \frac{(p_1 + p_2)^2 - 2p_1p_2}{p_1p_2} = \frac{256 - 2}{1} = 254$$

16. (b) : Given, $f''(x) = 6(x - 1)$ ...(i)

Integrating (i) on both sides, we get

$f'(x) = 3(x-1)^2 + c_1$ ...(ii)

$\Rightarrow\ 3 = 3 + c_1\ \{\because f(x) = y = 3x+5\ \therefore f'(x) = 3\ \forall x \in R\}$

$\Rightarrow\ c_1 = 0$

Integrating (ii) on both sides, we get

$f(x) = (x-1)^3 + c_2$

$\Rightarrow\ 1 = (2-1)^3 + c_2 \Rightarrow c_2 = 0$

$\therefore\ f(x) = (x-1)^3$

17. (c) : Given, $f(x) = \begin{cases} \dfrac{\sin^3 x^2}{x} & \forall x \neq 0 \\ 0 & \forall x = 0 \end{cases}$

Now, $\lim\limits_{x\to 0} \dfrac{\sin^3 x^2}{x} = \lim\limits_{x\to 0} \dfrac{\sin x^2}{x^2}(x\sin^2 x^2)$

$= \lim\limits_{x\to 0}\left(\dfrac{\sin x^2}{x^2}\right)\lim\limits_{x\to 0}(x\sin^2 x^2) = 0$

$\therefore\ \lim\limits_{x\to 0}\dfrac{\sin^3 x^2}{x} = 0 = f(0)$

So, $f(x)$ is continuous at $x = 0$

Now $Lf'(0) = \lim\limits_{h\to 0}\dfrac{f(0+h) - f(0)}{h} = \lim\limits_{h\to 0}\dfrac{\dfrac{\sin^3 h^2}{h} - 0}{h} = 0$

Similarly, $Rf'(0) = 0$

18. (d) : $f(x) = \displaystyle\int_{x^2}^{x^3}\frac{dt}{\log t}, x > 0$

$\Rightarrow\ f'(x) = \dfrac{1}{\log x^3}\cdot\dfrac{d(x^3)}{dx} - \dfrac{1}{\log x^2}\cdot\dfrac{d(x^2)}{dx}$

$= \dfrac{3x^2}{3\log x} - \dfrac{2x}{2\log x} = \dfrac{x^2 - x}{\log x} = \dfrac{x(x-1)}{\log x}$

Now, $f'(x) = 0$ gives $x(x-1) = 0 \Rightarrow x = 0, x = 1$

Let us consider $x = 1$, as $x > 0$

Hence, for $x < 1$, $f'(x) = (-\text{ve})(-\text{ve}) = (+\text{ve}) > 0$

and for $x > 1$, $f'(x) = (+\text{ve})(+\text{ve}) = (+\text{ve}) > 0$

$\therefore\ f'(x)$ does not change its sign in the intermediate neighborhood of $x = 1$

So $x = 1$ is neither the point of maxima or minima.

But $f'(x) > 0\ \forall\ x < 1$ and $x > 1$

Therefore, $f(x)$ is an increasing function $\forall\ x \in R$

19. (c) : For $y = 4x^2$ and $y = 3x$, the point of intersection of curves are $(0,0)$ and $\left(\dfrac{3}{4}, \dfrac{9}{4}\right)$.

Therefore, the area enclosed by the above curves is

$= \displaystyle\int_0^{3/4}(3x - 4x^2)dx = \left[\frac{3}{2}x^2 - \frac{4}{3}x^3\right]_0^{3/4} = \frac{9}{32} = A_1$

Again the area of the region enclosed by the curves $y = 4x^2$ and $y = |3x|$

$= 2\displaystyle\int_0^{3/4}(3x - 4x^2)dx = 2\times\left[\frac{3}{2}x^2 - \frac{4}{3}x^3\right]_0^{3/4} = 2\times\frac{9}{32} = A_2$

Therefore, $A_1 : A_2 = 1 : 2$

20. (c) : The given differential equation is

$$\frac{dt}{dx} = \frac{t\left(\dfrac{dg(x)}{dx}\right) - t^2}{g(x)}$$

$\Rightarrow\ \dfrac{dt}{dx} = t\dfrac{g'(x)}{g(x)} - \dfrac{t^2}{g(x)}\ \Rightarrow\ \dfrac{dt}{dx} - t\dfrac{g'(x)}{g(x)} = -\dfrac{t^2}{g(x)}$

$\Rightarrow\ -\dfrac{1}{t^2}\dfrac{dt}{dx} + \dfrac{1}{t}\dfrac{g'(x)}{g(x)} = \dfrac{1}{g(x)}$

$\Rightarrow\ \dfrac{dz}{dx} + z\dfrac{g'(x)}{g(x)} = \dfrac{1}{g(x)}\left[\text{Let } \dfrac{1}{t} = z \therefore -\dfrac{1}{t^2}\dfrac{dt}{dx} = \dfrac{dz}{dx}\right]$

The above equation is of type $\dfrac{dy}{dx} + Py = Q$

$\therefore\ \text{I.F.} = e^{\int\frac{g'(x)}{g(x)}dx} = g(x)$

So, solution is $zg(x) = \displaystyle\int\frac{1}{g(x)}g(x)dx = x + c$

$\Rightarrow\ \dfrac{1}{t}g(x) = x + c\ \Rightarrow\ t = \dfrac{g(x)}{x+c}$

21. (a) : Let $z = e^{i\theta}$ $\therefore$ $\arg z = \theta$

Also suppose that $\arg z_1 = \theta_1$ and $\arg z_2 = \theta_2$

$\therefore\ z_1Rz_2 \Leftrightarrow |\arg z_1 - \arg z_2| = \dfrac{2\pi}{3}$

$\therefore\ z_1Rz_2 \Leftrightarrow |\theta_1 - \theta_2| = \dfrac{2\pi}{3} \Leftrightarrow z_2Rz_1$

but $z_1 \neq z_2$ (if $z_1 = z_2$ then $\dfrac{2\pi}{3} = 0$ which is not true).

$\Rightarrow\ R$ is symmetric.

22. (c) : We have, $\lim\limits_{x\to 0}\dfrac{f(x)}{x} = 2$

$$\Rightarrow \lim_{x\to 0}\frac{\min\left\{\sin\sqrt{[m]}x, |x|\right\}}{x}=2$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin\sqrt{[m]}x}{x}=2 \quad \left[\because \sin\lambda x \le |x| \,\forall\, \lambda, x\right]$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin\sqrt{[m]}x}{\sqrt{[m]}x}\times\sqrt{[m]}=2$$

$$\Rightarrow \sqrt{[m]}=2 \Rightarrow [m]=4 \Rightarrow 4\le m<5$$

23. (c) : $\because \tan 3A=\dfrac{3\tan A-\tan^3 A}{1-3\tan^2 A}$

$$\Rightarrow \tan 30^\circ=\frac{3\tan 10^\circ-\tan^3 10^\circ}{1-3\tan^2 10^\circ} \text{ (Putting } A = 10^\circ)$$

$$\Rightarrow \frac{1}{\sqrt{3}}=\frac{3\tan 10^\circ-\tan^3 10^\circ}{1-3\tan^2 10^\circ}$$

$$\Rightarrow 1-3\tan^2 10^\circ=\sqrt{3}(3\tan 10^\circ-\tan^3 10^\circ)$$

$$\Rightarrow 1 + 9\tan^4 10^\circ - 6\tan^2 10^\circ = 3(9\tan^2 10^\circ + \tan^6 10^\circ - 6\tan^4 10^\circ)$$

$$\Rightarrow 3\tan^6 10^\circ - 27\tan^4 10^\circ + 33\tan^2 10^\circ = 1$$

24. (c) : Let θ be the angle between the line and plane.

$$\therefore \sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

$$[\text{where } \vec{b}=2\hat{i}-3\hat{j}+2\hat{k}, \vec{n}=3\hat{i}-2\hat{j}+5\hat{k}]$$

$$\Rightarrow \theta=\sin^{-1}\left(\frac{22}{\sqrt{646}}\right)$$

25. (b) : Given line is $\dfrac{x-3}{2}=\dfrac{y-4}{5}=\dfrac{z-6}{7}$.

Thus direction ratios are 2, 5, 7 (say a_1, b_1, c_1).
Given plane is $2x - 5y + 3z = 9$. Thus the direction ratios to the normal to the plane are 2, –5, 3 (say a_2, b_2, c_2)
Since $a_1a_2 + b_1b_2 + c_1c_2 = 4 - 25 + 21 = 0$, therefore the line is parallel to the plane.

26. (b) : Given determinant shows the volume of parallelopiped made by vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

$$\therefore \begin{vmatrix}\alpha_1 & \alpha_2 & \alpha_3\\ \beta_1 & \beta_2 & \beta_3\\ \gamma_1 & \gamma_2 & \gamma_3\end{vmatrix}=|\vec{a}||\vec{b}\times\vec{c}|\sin\frac{\pi}{3}$$

$$=|\vec{a}||\vec{b}||\vec{c}|\sin\frac{\pi}{3}\sin\frac{\pi}{6}=\frac{\sqrt{3}|\vec{b}||\vec{c}|}{\sqrt{2}} \quad [\because |\vec{a}|=2\sqrt{2}]$$

27. (c) : Mean $\bar{x}=\dfrac{a+(a+d)+...+(a+2nd)}{2n+1}=a+nd$

x_i	$\lvert x_i-\bar{x}\rvert$	$\lvert x_i-\bar{x}\rvert^2$
a	nd	n^2d^2
$a + d$	$(n - 1)d$	$(n - 1)^2d^2$
. . .	. . .	. . .
$a + (n - 2)d$	$2d$	$4d^2$
$a + (n - 1)d$	d	d^2
$a + nd$	0	0
$a + (n + 1)d$	$1d$	d^2
$a + (n + 2)d$	$2d$	$4d^2$
. . .	. . .	. . .
$a + 2nd$	nd	n^2d^2

$$\sum|x_i-\bar{x}|=2dn\left(\frac{n+1}{2}\right)=n(n+1)d$$

$$\sum|x_i-\bar{x}|^2=2d^2[1^2+2^2+3^2.....+n^2]=2d^2\,\frac{n(n+1)(2n+1)}{6}$$

Now, M.D. $=\dfrac{1}{n}\sum|x_i-\bar{x}|=\dfrac{n(n+1)d}{2n+1}$

$$\text{S.D.} = \sqrt{\frac{1}{n}\sum|x_i - \bar{x}|^2} = \sqrt{\frac{2d^2 \cdot n(n+1)(2n+1)}{6(2n+1)}}$$

$$= \sqrt{\frac{n(n+1)d^2}{3}} = \sqrt{\frac{n(n+1)}{3}} \cdot d$$

28. (a) : The given expression is

$$\sqrt{4\sin^4\theta + \sin^2 2\theta + 4\cos^2\left(\frac{\pi}{4} - \frac{\theta}{2}\right)}$$

$$= \sqrt{4\sin^2\theta(\sin^2\theta + \cos^2\theta) + 2\left(1 + \cos\left(\frac{\pi}{2} - \theta\right)\right)}$$

$$= \sqrt{4\sin^2\theta} + 2(1 + \sin\theta) = -2\sin\theta + 2 + 2\sin\theta$$

$$= 2 \quad \left[\text{as } \theta \in \left(\pi, \frac{3\pi}{2}\right)\right]$$

29. (c) : In the triangle ABC, where $\angle C = \theta$, we have $b\cos\theta = a$.

$\therefore \quad b\cos\theta + b = 4 \quad \Rightarrow \quad b = \dfrac{4}{1 + \cos\theta}$

$\therefore \quad a = \dfrac{4\cos\theta}{1 + \cos\theta}$

$\therefore \quad \text{Area } (\Delta) = \dfrac{1}{2} ba \sin\theta$

$$= \frac{1}{2} \cdot \frac{4}{1 + \cos\theta} \cdot \frac{4\cos\theta}{1 + \cos\theta} \cdot \sin\theta = \frac{4\sin 2\theta}{(1 + \cos\theta)^2}$$

$$\frac{d\Delta}{d\theta} = 4 \cdot \frac{2\cos 2\theta(1 + \cos\theta)^2 + \sin 2\theta \cdot 2(1 + \cos\theta)\sin\theta}{(1 + \cos\theta)^4}$$

$\therefore \quad \dfrac{d\Delta}{d\theta} = 0 \Rightarrow \cos 2\theta \cdot (1 + \cos\theta) + \sin 2\theta \cdot \sin\theta = 0$

$\Rightarrow \quad \cos 2\theta + \cos\theta = 0 \quad \Rightarrow \quad \cos 2\theta = -\cos\theta = \cos(\pi - \theta)$

$\Rightarrow \quad \theta = \dfrac{\pi}{3}$

$\therefore \quad \Delta$ is maximum when $\theta = \pi/3$ (It cannot be minimum, since in the given situation minimum is 0)

30. (b) : Let the edge of the cube be x unit.

Therefore, volume $V = x^3$ and percent error in measuring $x = \dfrac{dx}{x} \times 100 = k$.

The percent error in measuring volume $V = \dfrac{dV}{V} \times 100$

$\Rightarrow \quad \dfrac{dV}{V} = \dfrac{3x^2 dx}{x^3} = 3\dfrac{dx}{x}$

$\therefore \quad \dfrac{dV}{V} \times 100 = 3\dfrac{dx}{x} \times 100 = 3k$

◈◈

PRACTICE PAPER 2018
JEE MAIN

Exam Dates
OFFLINE : 8th April
ONLINE : 15th & 16th April

1. If $\text{Re}\left(\frac{z-8i}{z+6}\right)=0$, then z lies on the curve
(a) $x^2 + y^2 + 6x - 8y = 0$
(b) $4x - 3y + 24 = 0$
(c) $x^2 + y^2 - 8 = 0$ (d) none of these

2. If the ratio of the roots of $ax^2 + 2bx + c = 0$ is same as the ratio of the $px^2 + 2qx + r = 0$, then
(a) $\frac{2b}{ac}=\frac{q^2}{pr}$ (b) $\frac{b}{ac}=\frac{q}{pr}$
(c) $\frac{b^2}{ac}=\frac{q^2}{pr}$ (d) none of these

3. If A is 3 order square matrix such that $|A| = 2$ then $|\text{adj (adj (adj } A))|$ is
(a) 512 (b) 256
(c) 64 (d) none of these

4. If the matrices, A, B, $(A + B)$ are non-singular, then $[A(A + B)^{-1}B]^{-1}$ is
(a) $A^{-1}B^{-1}$ (b) $B^{-1} + A^{-1}$
(c) $B^{-1}A^{-1}$ (d) none of these

5. Given $a=\frac{x}{y-z}$, $b=\frac{y}{z-x}$ and $c=\frac{z}{x-y}$, where x, y and z are not all zero, then the value of $ab + bc + ca$
(a) 0 (b) 1
(c) -1 (d) none of these

6. Total number of 6-digit numbers that can be formed, having the property that every succeeding digit is greater than the preceding digit, is equal to
(a) ${}^9C_3$ (b) ${}^{10}C_3$ (c) ${}^9P_3$ (d) ${}^{10}P_3$

7. The integer which divides the number $101^{100} - 1$, is
(a) 100 (b) 1000
(c) 10,000 (d) 100,000

8. The sum of all three digit natural numbers, which are divisible by 7 is
(a) 60363 (b) 70336
(c) 140672 (d) none of these

9. Range of $f(x)=\sin^{-1}\left(\sqrt{x^2+x+1}\right)$ is
(a) $\left(0, \frac{\pi}{2}\right]$ (b) $\left(0, \frac{\pi}{3}\right]$
(c) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

10. If $f(x)$ is odd function and $f(1) = a$, and $f(x + 2) = f(x) + f(2)$ then the value of $f(3)$ is
(a) $6a$ (b) 0 (c) $9a$ (d) $3a$

11. $\lim_{x\to 1}\left[\text{cosec}\frac{\pi x}{2}\right]^{1/1-x}$ is equal to (where [.] represents the greatest integer function)
(a) 0 (b) 1
(c) ∞ (d) does not exist

12. Let $g(x)$ be a polynomial of degree one and $f(x)$ be defined by $f(x)=\begin{cases} g(x), & x\le 0 \\ |x|^{\sin x}, & x>0 \end{cases}$. If $f(x)$ is continuous satisfying $f'(1) = f(-1)$, then $g(1)$ is
(a) $(1 + \sin 1)x + 1$ (b) $(1 - \sin 1)x + 1$
(c) $(1 - \sin 1)x - 1$ (d) $(1 + \sin 1)x - 1$

13. If $f(0) = 0$, $f'(0) = 2$, then the derivative of $y = f(f(f(f(x))))$ at $x = 0$ is
(a) 2 (b) 8 (c) 16 (d) 4

14. The values of a if the curves $\frac{x^2}{a^2}+\frac{y^2}{4}=1$ and $y^3 = 16x$ cuts orthogonally
(a) $\pm\frac{1}{2}$ (b) $\pm\frac{2}{\sqrt{3}}$ (c) $\pm\frac{\sqrt{3}}{4}$ (d) $\pm\sqrt{3}$

15. If $3(a + 2c) = 4(b + 3d)$, then the equation $ax^3 + bx^2 + cx + d = 0$ will have
(a) no real solution
(b) at least one real root in $(-1, 0)$
(c) at least one real root in $(0, 1)$
(d) none of these

Sanjay Singh Mathematics Classes, Chandigarh, Ph : 9888228231, 9216338231

16. $\int x^5(1+x^3)^{2/3}dx = A(1+x^3)^{8/3} + B(1+x^3)^{5/3} + c$ then

(a) $A=\frac{1}{4}, B=\frac{1}{5}$ (b) $A=\frac{1}{8}, B=-\frac{1}{5}$

(c) $A=-\frac{1}{8}, B=\frac{1}{5}$ (d) none of these

17. If $\int_0^1 \cot^{-1}(1-x+x^2)dx = \lambda\int_0^1 \tan^{-1} x dx$ then 'λ' is equal to

(a) 1 (b) 2 (c) 3 (d) 4

18. The curve for which the normal at any point (x, y) and the line joining origin to that point form an isosceles triangle with the x-axis as base is

(a) an ellipse

(b) a rectangular hyperbola

(c) a circle (d) none of these

19. The points (a, b), (c, d) and $\left(\frac{kc+la}{k+l}, \frac{kd+lb}{k+l}\right)$ are

(a) vertices of an equilateral triangle

(b) vertices of an isosceles triangle

(c) vertices of a right angled triangle

(d) collinear

20. The straight line $ax + by + c = 0$ where $abc \neq 0$ will pass through the first quadrant if

(a) $ac > 0$, $bc > 0$ (b) $ac > 0$ and $bc < 0$

(c) $bc > 0$ and $ac > 0$ (d) $ac < 0$ and $bc < 0$

21. The circles having radii r_1 and r_2 intersect orthogonally. Length of their common chord is

(a) $\frac{\sqrt{r_1^2+r_2^2}}{2r_1r_2}$ (b) $\frac{\sqrt{r_1^2+r_2^2}}{r_1r_2}$

(c) $\frac{r_1r_2}{\sqrt{r_1^2+r_2^2}}$ (d) $\frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}}$

22. Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point $A(h, 0)$. If triangle OA_1A_2 is equilateral then possible values of h is

(a) 26 (b) 24

(c) 28 (d) none of these

23. A straight line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets the plane $\vec{r}\cdot\vec{n} = 0$ in P. The position vector of P is

(a) $\vec{a} + \frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$ (b) $\vec{b} - \frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{a}$

(c) $\vec{a} - \frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$ (d) none of these

24. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar vectors, then

(a) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{b} & \vec{c} & \vec{a} \\ \vec{c} & \vec{a} & \vec{b} \end{vmatrix} = \vec{0}$ (b) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \end{vmatrix} = \vec{0}$

(c) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & \vec{c}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{b} \end{vmatrix} = \vec{0}$ (d) $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{c} & \vec{c}\cdot\vec{b} \end{vmatrix} = \vec{0}$

25. If the letters of the word 'REGULATIONS' be arranged at random, then the probability that there will be exactly 4 letters between the R and the E is

(a) $\frac{{}^9C_4\, 6!}{11!}$ (b) $\frac{{}^9C_5\, 6!2!}{12!}$

(c) $\frac{{}^9P_4\, 6!2!}{11!}$ (d) $\frac{{}^9C_4\, 6!2!4!}{11!}$

26. Let $z = x + iy$ be a complex numbers where x and y are integers. Then area of the rectangle whose vertices are roots of $z\bar{z}^3 + \bar{z}z^3 = 738$ is

(a) 80 (b) 48 (c) 56 (d) 96

27. In a hurdle race, a runner has probability p of jumping over a specific hurdle. Given that in 5 trails, the runner succeeded 3 times, the conditional probability that the runner had succeeded in the first trial is

(a) 3/5 (b) 2/5 (c) 1/5 (d) 4/5

28. The points of intersection of the two ellipses
$x^2 + 2y^2 - 6x - 12y + 23 = 0$ and
$4x^2 + 2y^2 - 20x - 12y + 35 = 0$.

(a) lie on a circle centred at (8/3, 3) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(b) lie on a circle centred at (–8/3, 3) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(c) lie on a circle centred at (8, 9) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$

(d) are not cyclic

29. Equation of a plane which passes through the point of intersection of lines $\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2}$ and $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and at greatest distance from the origin is

(a) $7x + 2y + 4z = 54$ (b) $4x + 3y + 5z = 50$

(c) $3x + 4y + 5z = 49$ (d) $x + y + z = 12$

30. If $f(x)=\begin{cases}\sin^{-1}(\sin x), & x>0\\ \dfrac{\pi}{2}, & x=0\\ \cos^{-1}(\cos x), & x<0\end{cases}$, then

(a) $x = 0$ is a point of maxima
(b) $x = 0$ is a point of minima
(c) $x = 0$ is point of intersection
(d) none of these

31. If the angle of intersection of the circles $x^2 + y^2 + x + y = 0$ and $x^2 + y^2 + x - y = 0$ is θ, then equation of the line passing through (1, 2) and making an angle θ with the y-axis is

(a) $x = 1$ (b) $y = 2$
(c) $x + y = 3$ (d) $x - y = 3$

32. If $f(n)=\sum_{k=1}^{n}\sum_{j=k}^{n}({}^nC_j)({}^jC_k)$ the value of $f(6)$

(a) 660 (b) 665
(c) 670 (d) none of these

33. Let f be a continuous and differentiable function in (a, b), $\lim_{x\to a^+} f(x)\to\infty$ and $\lim_{x\to b^-} f(x)\to-\infty$. If $f'(x)+f^2(x)\ge -1$ for $a < x < b$, then

(a) $b - a \le \pi$ (b) $b - a \ge \pi$
(c) $b - a = \pi$ (d) none of these

34. If $\dfrac{\tan 3A}{\tan A}=a$, then $\dfrac{\sin 3A}{\sin A}$ is equal to

(a) $\dfrac{2a}{a+1}$ (b) $\dfrac{2a}{a-1}$ (c) $\dfrac{a}{a+1}$ (d) $\dfrac{a}{a-1}$

35. The value of $\int_e^{\pi^2}[\log_\pi x]d(\log_e x)$ (where [.] denotes greatest integer function) is

(a) $2\log_e\pi$ (b) $\log_e\pi$
(c) 1 (d) 0

SOLUTIONS

1. **(a) :** Let $z = x + iy$. Then

$$\frac{z-8i}{z+6}=\frac{x+(y-8)i}{(x+6)+iy}=\frac{(x+(y-8)i)(x+6-iy)}{(x+6)^2+y^2}$$

$$\frac{(x^2+6x+y^2-8y)+i(xy-8x+6y-48-xy)}{(x+6)^2+y^2}$$

$$\Rightarrow \operatorname{Re}\left(\frac{z-8i}{z+6}\right)=0 \Rightarrow x^2 + y^2 + 6x - 8y = 0$$

2. **(c) :** Let roots of the equation $ax^2 + 2bx + c = 0$ are α and β and roots of the equation $px^2 + 2qx + r = 0$ are γ and δ.

Given, $\dfrac{\alpha}{\beta}=\dfrac{\gamma}{\delta} \Rightarrow \dfrac{\alpha}{\gamma}=\dfrac{\beta}{\delta}$

$$\Rightarrow \frac{\alpha+\beta}{\gamma+\delta}=\sqrt{\frac{\alpha\beta}{\gamma\delta}} \Rightarrow \frac{-\frac{2b}{a}}{-\frac{2q}{p}}=\sqrt{\frac{\frac{c}{a}}{\frac{r}{p}}} \Rightarrow \frac{b^2}{ac}=\frac{q^2}{pr}$$

3. **(b) :** We know that $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

$\Rightarrow |\text{adj}(\text{adj }(\text{adj } A))| = |\text{adj } A|^{(n-1)^2} = |A|^{(n-1)^3}$
$= 2^8 = 256$

4. **(b) :** $[A(A + B)^{-1}B]^{-1} = B^{-1}((A + B)^{-1})^{-1}A^{-1}$
$= B^{-1}(A + B)A^{-1}$ $[\because (A^{-1})^{-1} = A]$
$= (B^{-1}A + I)A^{-1} = B^{-1}I + IA^{-1} = B^{-1} + A^{-1}$

5. **(c) :** $a = x/(y - z) \Rightarrow x - ay + az = 0$
$b = y/(z - x) \Rightarrow bx + y - bz = 0$
$c = z/(x - y) \Rightarrow -cx + cy + z = 0$

As x, y, z are not all zero, thus the above system has a non-trivial solution

$$\therefore \Delta=\begin{vmatrix}1 & -a & a\\ b & 1 & -b\\ -c & c & 1\end{vmatrix}=0 \Rightarrow 1 + ab + bc + ca = 0$$

6. **(a) :** $x_1 < x_2 < x_3 < x_4 < x_5 < x_6$, when the number is $x_1x_2x_3x_4x_5x_6$

Clearly, no digit can be zero.

Also, all the digits are distinct. So, let us first select six digits from the list of digits 1, 2, 3, 4, 5, 6, 7, 8, 9 which can be done in ${}^9C_6$ ways.

After selecting these digits, they can be put only in one order.

Thus, total number of such numbers = ${}^9C_6 \times 1 = {}^9C_3$

7. **(c) :** $(1 + 100)^{100} = 1 + 100\cdot100$

$$+\frac{100\cdot99}{1\cdot2}\cdot(100)^2+\frac{100\cdot99\cdot98}{1\cdot2\cdot3}(100)^3+......$$

$\Rightarrow (101)^{100} - 1$

$$=100\cdot100\left[1+\frac{100\cdot99}{1\cdot2}+\frac{100\cdot99\cdot98}{1\cdot2\cdot3}\cdot100+.......\right]$$

From above it is clear that $(101)^{100} - 1$ is divisible by $(100)^2 = 10{,}000$

8. **(b) :** The sequence of three-digit numbers which are divisible by 7 are 105, 112, 119, …., 994.

Clearly, it is an A.P. with first term $a = 105$ and common difference $d = 7$

Let there be n terms in this sequence. Then, $a_n = 994$

$\Rightarrow\ a + (n-1)d = 994$

$\Rightarrow\ 105 + (n-1)\times 7 = 994 \ \Rightarrow\ n = 128$

$\therefore$ Required sum $= \frac{n}{2}[a_1 + a_n] = \frac{128}{2}[105 + 994] = 70336$

9. (c) : For domain point of view, $0 \le x^2 + x + 1 \le 1$,

but $x^2 + x + 1 \ge \frac{3}{4} \Rightarrow \frac{\sqrt{3}}{2} \le \sqrt{x^2+x+1} \le 1$

$\Rightarrow\ \frac{\pi}{3} \le \sin^{-1}(\sqrt{x^2+x+1}) \le \frac{\pi}{2}$

10. (d) : Put $x = -1$ in $f(x+2) = f(x) + f(2)$

$\Rightarrow f(1) = f(-1) + f(2) \ \Rightarrow f(1) = -f(1) + f(2)$

$\Rightarrow f(2) = 2a$

Now put $x = 1 \Rightarrow f(3) = f(1) + f(2) = a + 2a = 3a$

11. (b) : $\operatorname{cosec}\frac{\pi x}{2} \to 1$ when $x \to 1$

$\therefore$ Limit = 1

12. (b) : $f(0^+) = \lim\limits_{x\to 0^+} |x|^{\sin x} = 1$

$f(0^-) = g(0) = 1$

Let $g(x) = ax + b$

$\Rightarrow b = 1 \ \Rightarrow\ g(x) = ax + 1$

For $x > 0$, $f'(x) = e^{\sin x \ln(|x|)}\left[\cos x \ln(|x|) + \frac{\sin x}{x}\right]$

$f'(1) = 1[0 + \sin 1] = \sin 1$

$f(-1) = -a + 1 \ \Rightarrow\ a = 1 - \sin 1$

$g(1) = (1 - \sin 1)x + 1$

13. (c) : $y'(x) = f'(f(f(f(x))))\, f'(f(f(x)))\, f'(f(x))f'(x)$

$\Rightarrow y'(0) = f'(f(f(f(0))))\, f'(f(f(0)))\, f'(f(0))\, f'(0)$

$= f'(f(f(0)))\, f'(f(0))\, f'(0)\, f'(0)$

$= f'(f(0))\, f'(0)\, f'(0)\, f'(0)$

$= f'(0)\, f'(0)\, f'(0)\, f'(0) = (f'(0))^4 = 2^4 = 16$

14. (b) : The two curves are $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ ... (i)

and $y^3 = 16x$ ... (ii)

Diff. (i), we get $dy/dx = -4x/(a^2y) = m_1$

Diff. (ii), we get $dy/dx = 16/(3y^2) = m_2$

The two curves cut orthogonally

$\therefore\ m_1m_2 = -1$

$\Rightarrow\ [-4x/(a^2y)]\ [16/(3y^2)] = -1 \Rightarrow 64x = 3a^2y^3$

$\Rightarrow\ 64x = 3a^2\cdot 16x$ [using (ii)]

$\Rightarrow\ a^2 = 4/3 \Rightarrow a = \pm\frac{2}{\sqrt{3}}.$

15. (b) : Let $f(x) = \frac{ax^4}{4} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx$ which is continuous and differentiable.

$f(0) = 0,\ f(-1) = \frac{a}{4} - \frac{b}{3} + \frac{c}{2} - d$

$= \frac{1}{4}(a + 2c) - \frac{1}{3}(b + 3d) = 0$

So, according to Rolle's theorem, there exist at least one root of $f'(x) = 0$ in $(-1, 0)$.

16. (b) : Let $I = \int x^5(1+x^3)^{2/3}dx$

Let $1 + x^3 = t^2$ and $3x^2\,dx = 2t\,dt$

$\therefore\ \int x^5(1+x^3)^{2/3}dx = \int x^3(1+x^3)^{2/3}x^2\,dx$

$= \int (t^2-1)(t^2)^{2/3}\frac{2}{3}t\,dt = \frac{2}{3}\int (t^2-1)t^{7/3}dt$

$= \frac{2}{3}\int (t^{13/3} - t^{7/3})dt = \frac{2}{3}\left\{\frac{3}{16}t^{16/3} - \frac{3}{10}t^{10/3}\right\} + c$

$= \frac{1}{8}(1+x^3)^{8/3} - \frac{1}{5}(1+x^3)^{5/3} + c$

17. (b) : $\int_0^1 \cot^{-1}(1 - x + x^2)dx = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$

$= \int_0^1 \tan^{-1}\left(\frac{x + (1-x)}{1 - x(1-x)}\right)dx$

$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$

$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}[1-(1-x)]dx = 2\int_0^1 \tan^{-1}x\,dx$

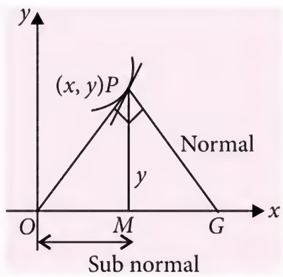
$\Rightarrow\ \lambda = 2$

18. (b) : It is given that the triangle OPG is an isosceles triangle.

Therefore,

$OM = MG$ = subnormal

$\Rightarrow\ x = y\frac{dy}{dx} \ \Rightarrow\ xdx = ydy$



On integration, we get

$x^2 - y^2 = C$,

which is a rectangular hyperbola.

19. (d) : The given points are collinear because the point $\left(\frac{kc+la}{k+l}, \frac{kd+lb}{k+l}\right)$ divides the points (a, b) and (c, d) in the ratio of $k : l$.

20. (d) : If the line cut x and y-axis at A and B then

$$A \equiv \left(-\frac{c}{a}, 0\right),\ B \equiv \left(0, -\frac{c}{b}\right)$$

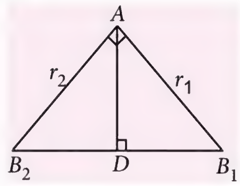
Line will pass through the first quadrant if

$$-\frac{c}{a} > 0 \text{ and } -\frac{c}{b} > 0 \Rightarrow ac < 0 \text{ and } bc < 0$$

21. (d) : Let $\angle AB_1B_2 = \theta$

$\Rightarrow AD = r_1 \sin\theta$ and $AD = r_2 \cos\theta$

$$\Rightarrow AD^2\left(\frac{1}{r_1^2} + \frac{1}{r_2^2}\right) = 1$$



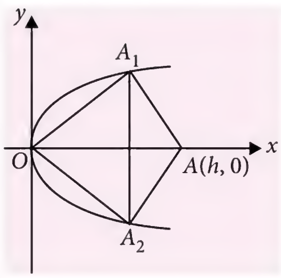
$$\Rightarrow AD = \frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$

Thus length of common chord $= \dfrac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$

22. (c) : Let $A_1 \equiv (2t_1^2, 4t_1)$, $A_2 \equiv (2t_1^2, -4t_1)$

Clearly, $\angle A_1OA = \pi/6$

$$\Rightarrow \frac{2}{t_1} = \frac{1}{\sqrt{3}} \Rightarrow t_1 = 2\sqrt{3}$$



Equation of normal at A_1 is

$y = -t_1x + 4t_1 + 2t_1^3$

$\Rightarrow h = 4 + 2t_1^2$

$= 4 + 2 \cdot 12 = 28$

23. (c) : The straight line $\vec{r} = \vec{a} + \lambda\vec{b}$ meets the plane $\vec{r}\cdot\vec{n} = 0$ in P for which λ is given by

$$(\vec{a} + \lambda\vec{b})\cdot\vec{n} = 0 \Rightarrow \lambda = -\frac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}$$

Thus, the position vector of P is $\vec{r} = \vec{a} - \dfrac{\vec{a}\cdot\vec{n}}{\vec{b}\cdot\vec{n}}\vec{b}$

[Putting the value of λ in $\vec{r} = \vec{a} + \lambda\vec{b}$]

24. (b) : Since $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, there exists (x, y, z not all zero) such that

$$x\vec{a} + y\vec{b} + z\vec{c} = 0 \quad \text{... (i)}$$

Multiply by $\vec{a}$ and $\vec{b}$ scalarly, we get

$$x(\vec{a}\cdot\vec{a}) + y(\vec{a}\cdot\vec{b}) + z(\vec{a}\cdot\vec{c}) = 0 \quad \text{... (ii)}$$

and $x(\vec{b}\cdot\vec{a}) + y(\vec{b}\cdot\vec{b}) + z(\vec{b}\cdot\vec{c}) = 0$

Eliminating x, y and z from (i), (ii) and (iii), we get

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c} \end{vmatrix} = 0$$

25. (c) : Total number of ways of arranging 11 letters $= (11)!$ ... (i)

Selection of 4 letters to be placed between R and E from remaining $9 = {}^9C_4$.

These 4 letters can be permuted in 4! ways.

Now permutation of $(R\ \underbrace{\bullet\bullet\bullet\bullet}_{4\text{ letters}}\ E)\ \underbrace{\bullet\bullet\bullet\bullet\bullet}_{5\text{ letters}} = 6!$

R and E can be interchanged in 2! ways

$\therefore$ Number of favourable ways $= {}^9C_4 \cdot 6! \cdot 2!\ 4!$ ... (ii)

Hence, the required probability is $\dfrac{{}^9C_4\, 6!2!4!}{11!}$

26. (a) : We have $z\bar{z}^3 + \bar{z}z^3 = 738$

$\Rightarrow z\bar{z}(\bar{z}^2 + z^2) = 738$

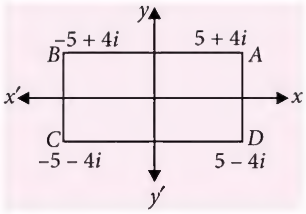
$\Rightarrow (x^2 + y^2)[(x - iy)^2 + (x + iy)^2] = 738$

$\Rightarrow (x^2 + y^2)(2x^2 - 2y^2) = 738$

$\Rightarrow x^4 - y^4 = 369$

$\Rightarrow x^4 \geq 369$

$\Rightarrow |x| \geq 4$



Let us try $x = \pm 5$

$\Rightarrow 625 - 369 = y^4$

$\Rightarrow y = \pm 4$

Thus, roots of $z\bar{z}^3 + \bar{z}z^3 = 738$ are $\pm 5 \pm 4i$.

Area of the rectangle $ABCD$ whose vertices are $\pm 5 \pm 4i$ is $(10)\ (8) = 80$

27. (a) : Let A denote the event that the runner succeeds exactly 3 times out of five and B denote the event that the runner succeeds on the first trial.

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

But $P(B \cap A) = P$ (succeeding in the first trial and exactly twice in other four trials)

$= p({}^4C_2\, p^2(1 - p)^2) = 6p^3(1 - p)^2$

And $P(A) = {}^5C_3\, p^3\,(1 - p)^2 = 10p^3(1 - p)^2$

$$\text{Thus, } P(B|A) = \frac{6p^3(1-p)^2}{10p^3(1-p)^2} = \frac{3}{5}$$

28. (a) : Equation of any curve passing through the intersection of the given ellipses is given by

$4x^2 + 2y^2 - 20x - 12y + 35$
$+ \lambda(x^2 + 2y^2 - 6x - 12y + 23) = 0$

which represents a circle if

$4 + \lambda = 2 + 2\lambda \Rightarrow \lambda = 2$

and the equation of the circle is thus,

$6x^2 + 6y^2 - 32x - 36y + 81 = 0$

$$\Rightarrow \quad x^2+y^2-\left(\frac{16}{3}\right)x-6y+\frac{81}{6}=0$$

Centre of the circle is (8/3, 3)

and the radius is $\sqrt{\left(\frac{8}{3}\right)^2+(3)^2-\frac{81}{6}}$

$$=\sqrt{\frac{128+162-243}{18}}=\frac{1}{3}\sqrt{\frac{47}{2}}$$

29. (b) : Any point on the first line is $P(3\lambda + 1, \lambda + 2, 2\lambda + 3)$ and on the second line is $Q(\mu + 3, 2\mu + 1, 3\mu + 2)$. P and Q represent the same point if $\lambda = \mu = 1$ and the point of intersection of the given lines is $P(4, 3, 5)$. The planes given in (a), (b), (c) and (d) all pass through P, the plane at greatest distance is one which is at a distance equal to OP from the origin. So the distance of the plane from the origin is

$$\sqrt{4^2+3^2+5^2}=\sqrt{50}$$

which is satisfied by the plane is $4x + 3y + 5z = 50$.

30. (a) : $f(0) = \pi/2$, $f(0^+) = 0$, $f(0^-) = 0$.
Hence, $x = 0$ is the point of maxima.

31. (b) : Let A and B be the centres and r_1 and r_2 the radii of the two circles, then

$$A\equiv\left(-\frac{1}{2},-\frac{1}{2}\right),\ B=\left(-\frac{1}{2},\frac{1}{2}\right),\ r_1=\frac{1}{\sqrt{2}},\ r_2=\frac{1}{\sqrt{2}}$$

$$\cos\theta=\frac{r_1^2+r_2^2-AB^2}{2r_1r_2}=\frac{\frac{1}{2}+\frac{1}{2}-1}{2\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}}=0$$

$\therefore \quad \theta = \pi/2$

Required line is parallel to the x-axis and since it passes through (1, 2), its equation will be $y = 2$.

32. (b) : $f(n)=\sum_{j=1}^{n}({}^nC_j)({}^jC_1)+\sum_{j=2}^{n}({}^nC_j)({}^jC_2)+....$

$$+\sum_{j=n-1}^{n}({}^nC_j)({}^jC_{n-1})+({}^nC_n)({}^nC_n)$$

$= ({}^nC_1)({}^1C_1) + ({}^nC_2)({}^2C_1 + {}^2C_2) + ({}^nC_3)({}^3C_1 + {}^3C_2 + {}^3C_3) + ... + ({}^nC_n)({}^nC_1 + {}^nC_2 + ... + {}^nC_n)$

$= ({}^nC_1)(2 - 1) + ({}^nC_2)(2^2 - 1) + ({}^nC_3)(2^3 - 1) + ... + {}^nC_n\,(2^n - 1)$

$= \{(2 + 1)^n - 1\} - \{(1 + 1)^n - 1\} = 3^n - 2^n$

33. (b) : We have, $f'(x) + f^2(x) \geq -1$

$$\Rightarrow \quad \frac{f'(x)}{1+f^2(x)}+1\geq 0 \text{ for } x\in(a, b) \qquad ... \text{(i)}$$

$$\frac{d}{dx}(\tan^{-1}f(x)+x)=\frac{f'(x)}{1+(f(x))^2}+1\geq 0 \qquad \text{((from (i))}$$

$\Rightarrow$ $h(x) = \tan^{-1} f(x) + x$, is a non-decreasing function in the interval (a, b).

$$\Rightarrow \quad \lim_{x\to a} h(x)\leq \lim_{x\to b} h(x)$$

$$\Rightarrow \quad \lim_{x\to a^+}(\tan^{-1}f(x)+x)\leq \lim_{x\to b^-}(\tan^{-1}f(x)+x)$$

$$\Rightarrow \quad \frac{\pi}{2}+a\leq -\frac{\pi}{2}+b$$

Hence, $b - a \geq \pi$

34. (b) : Given, $\frac{\tan 3A}{\tan A}=a$

$$\Rightarrow \quad \frac{3\tan A-\tan^3 A}{\tan A(1-3\tan^2 A)}=a$$

$\Rightarrow$ $3 - \tan^2 A = a - 3a\tan^2 A$

$\Rightarrow$ $\tan^2 A(3a - 1) = a - 3$

$$\Rightarrow \quad \tan A=\pm\sqrt{\frac{a-3}{3a-1}}$$

Now, $\frac{\sin 3A}{\sin A}=\frac{3\sin A-4\sin^3 A}{\sin A}$

$$=3-4\sin^2 A=3-4\left(\frac{a-3}{4(a-1)}\right)$$

$$=\frac{3a-3-a+3}{(a-1)}=\frac{2a}{(a-1)}$$

35. (b) : Let $\log_e x = t$

$$\therefore \quad I=\int_1^{\log_e \pi^2}[\log_\pi e^t]dt$$

$$\Rightarrow \quad I=\int_1^{\log_e \pi^2}[t\log_\pi e]dt$$

$$=\int_1^{\log_e \pi}0\,dt+\int_{\log_e \pi}^{2\log_e \pi}1\,dt=\log_e \pi$$

◈◈

WB JEE PRACTICE PAPER 2018

Exam on 22nd April 2018

CATEGORY-I (Q. 1 to Q. 50)

Only one answer is correct. Correct answer will fetch full marks 1. Incorrect answer or any combination of more than one answer will fetch –¼ marks. No answer will fetch 0 marks.

1. The principal value of $\sin^{-1}\left[\tan\left(-\frac{5\pi}{4}\right)\right]$ is
(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

2. The coefficient of x^7 in $\frac{1-x}{(1+x)^2}$ is
(a) 12 (b) 15 (c) –15 (d) none of these

3. Let $f: N \to R$ be such that $f(1) = 1$ and $f(1) + 2f(2) + 3f(3) + ... + nf(n) = n(n+1)f(n)$, for all $n \in N, n \geq 2$, where N is the set of natural numbers and R is the set of real numbers. Then the value of $f(500)$ is
(a) 1000 (b) 500 (c) 1/500 (d) 1/1000

4. If $I_1 = \int_0^{\pi/2} x\sin x\, dx$ and $I_2 = \int_0^{\pi/2} x\cos x\, dx$, then which one of the following is true?
(a) $I_1 = I_2$ (b) $I_1 + I_2 = 0$
(c) $I_1 = \frac{\pi}{2} I_2$ (d) $I_1 + I_2 = \frac{\pi}{2}$

5. If the ratio of the sums of m and n terms of an A.P. is $m^2 : n^2$, then the ratio of its m^{th} and n^{th} terms is given by
(a) $(2m + 1) : (2n + 1)$ (b) $(2m - 1) : (2n - 1)$
(c) $m : n$ (d) $m - 1 : n - 1$

6. The domain of the function $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is
(a) $R - \{-1, -2\}$ (b) $R - \{-1, -2, 0\}$
(c) $(-3, -1) \cup (-1, \infty)$ (d) $(-3, \infty) - \{-1, -2\}$

7. There are 8 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is
(a) 8! (b) 16 (c) 255 (d) 2^8

8. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then A^{100} is equal to
(a) $100\,A$ (b) $2^{99} A$ (c) $2^{100} A$ (d) $99\,A$

9. If $y = \frac{1}{1+x+x^2}$, then $\frac{dy}{dx} =$
(a) $y^2(1 + 2x)$ (b) $\frac{-(1+2x)}{y^2}$
(c) $-y^2(1 + 2x)$ (d) $\frac{(1+2x)}{y^2}$

10. Numbers 1, 2, 3, ..., $2n$ ($n \in N$) are printed on $2n$ cards. The probability of drawing a number r is proportional to r. Then the probability of drawing an even number in one draw is
(a) $\frac{n+2}{n+3}$ (b) $\frac{n+1}{n+3}$ (c) $\frac{1}{2}$ (d) $\frac{n+1}{2n+1}$

11. Area bounded by $y = \log(x - 2)$, x-axis and $x = 4$ is
(a) 2log2 + 1 (b) log2 – 1
(c) log2 + 1 (d) 2log2 – 1

12. We define a binary relation ~ on the set of all 3 × 3 real matrices as $A \sim B$ if and only if there exist invertible matrices P and Q such that $B = PAQ^{-1}$. The binary relation ~ is
(a) neither reflexive nor symmetric
(b) reflexive and symmetric but not transitive
(c) symmetric and transitive but not reflexive
(d) an equivalence relation

13. If $P(x)$ is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $P(0) = 0$, $P(1) = 1$, and $P'(x) > 0\ \forall x \in [0,1]$, then
(a) $S = \phi$
(b) $S = ax + (1 - a)x^2\ \forall\ a \in (0,2)$
(c) $S = ax + (1 - a)\, x^2\ \forall\ a \in (0,\infty)$
(d) $S = ax + (1 - a)x^2\ \forall\ a \in (0,1)$

14. $\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\frac{4}{5}x^5+.......\text{ to }\infty=$

(a) $\frac{x}{1-x}+\log(1-x)$ (b) $\frac{x}{1+x}+\log(1+x)$

(c) $-\frac{x}{1+x}+\log(1+x)$ (d) none of these

15. If $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ a & b & -1\end{bmatrix}$ and I is the unit matrix of order 3, then $A^2 + 2A^4 + 4A^6$ is equal to

(a) $7A^8$ (b) $7A^7$ (c) $8I$ (d) $6I$

16. The co - efficient of x^4 in the expansion of $\frac{1-3x+x^2}{e^x}$ is

(a) $\frac{25}{24}$ (b) $\frac{24}{25}$ (c) $\frac{4}{25}$ (d) $\frac{5}{24}$

17. If $A=\begin{bmatrix}1 & 0 & 0\\ x & 1 & 0\\ x & x & 1\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ then $A^3 - 3A^2 + 3A$ is equal to

(a) $3I$ (b) I (c) $-I$ (d) $-2I$

18. The sum of $\frac{1}{1\cdot 2}-\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}-\frac{1}{4\cdot 5}+.....$ is

(a) $\log(2e)$ (b) $\log(4e)$

(c) $\log\left(\frac{2}{e}\right)$ (d) $\log\left(\frac{4}{e}\right)$

19. $\begin{vmatrix}\sin\alpha & \cos\alpha & \sin(\alpha+\delta)\\ \sin\beta & \cos\beta & \sin(\beta+\delta)\\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta)\end{vmatrix}=$

(a) 0 (b) 1

(c) $1 + \sin\alpha\sin\beta\sin\gamma$

(d) $1 - (\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)(\sin\gamma - \sin\alpha)$

20. The general solution of $\sin x-\cos x=\sqrt{2}$, for any integer 'n' is

(a) $2n\pi+\frac{3\pi}{4}$ (b) $n\pi$ (c) $(2n + 1)\pi$ (d) $2n\pi$

21. If n is a natural number, then

(a) $1^2+2^2+........+n^2<\frac{n^3}{3}$

(b) $1^2+2^2+........+n^2=\frac{n^3}{3}$

(c) $1^2 + 2^2 + ... + n^2 > n^3$

(d) $1^2+2^2+........+n^2>\frac{n^3}{3}$

22. If * is the operation defined by $a * b = a^b$ for $a, b \in N$, then $(2 * 3) * 2$ is equal to

(a) 81 (b) 512 (c) 216 (d) 64

23. The sum of $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+.........\text{ to }\infty$ is

(a) e (b) e^{-1} (c) $2e$ (d) $2e^{-1}$

24. If A and B are mutually exclusive events such that $P(A) = 0.25$, $P(B) = 0.4$, then $P(A^c \cap B^c)$ is equal to

(a) 0.45 (b) 0.55 (c) 0.9 (d) 0.35

25. Five horses are in race. Mr. A select two of the horse at random and bets on them. The probability that Mr. A selected the winning horse is

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $\frac{4}{5}$

26. If ω is an imaginary cube root of unity, then the value of $\begin{vmatrix}1 & \omega^2 & 1-\omega^4\\ \omega & 1 & 1+\omega^5\\ 1 & \omega & \omega^2\end{vmatrix}$ is

(a) -4 (b) $\omega^2 - 4$ (c) ω^2 (d) 4

27. The matrix product satisfies $[5\ 6\ 2]\cdot A^T = [4\ 8\ 1\ 7\ 8]$, where A^T denotes the transpose of the matrix A. Then the order of the matrix A equals to

(a) 1×2 (b) 5×1 (c) 3×5 (d) 5×3

28. The solution set of the inequation $\cos^{-1} x < \sin^{-1} x$ is

(a) $[-1, 1]$ (b) $\left[\frac{1}{\sqrt{2}},1\right]$ (c) $[0, 1]$ (d) $\left(\frac{1}{\sqrt{2}},1\right]$

29. If n is a positive integer, then $n^3 + 2n$ is divisible by

(a) 15 (b) 3 (c) 2 (d) 6

30. The value of $\cot^{-1} 9 + \operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}$ is

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) π

31. The value of $\int_0^4 |x-1|\,dx$ is

(a) $\frac{5}{2}$ (b) 5 (c) 4 (d) 1

32. The sum of the series $1+\frac{3^2}{2!}+\frac{3^4}{4!}+\frac{3^6}{6!}+...$ to ∞ is

(a) e^{-3} (b) e^3

(c) $\frac{1}{2}(e^3-e^{-3})$ (d) $\frac{1}{2}(e^3+e^{-3})$

33. Find which function does not obey mean value theorem in [0, 1].

(a) $f(x) = |x|$ (b) $f(x) = x|x|$

(c) $f(x)=\begin{cases} \dfrac{\sin x}{x}, x \neq 0 \\ 1, x=0 \end{cases}$ (d) none of these

34. If the events A and B are independent and if $P(A^c) = \dfrac{2}{3}$, $P(B^c) = \dfrac{2}{7}$ then $P(A \cap B)$ is equal to

(a) $\dfrac{4}{21}$ (b) $\dfrac{3}{21}$ (c) $\dfrac{5}{21}$ (d) $\dfrac{1}{21}$

35. Find the coefficient independent of x in the expansion of $(1+x+2x^3)\left(\dfrac{3}{2}x^2 - \dfrac{1}{3x}\right)^9$

(a) $\dfrac{15}{54}$ (b) $\dfrac{11}{54}$ (c) $\dfrac{12}{54}$ (d) $\dfrac{17}{54}$

36. A sphere increases its volume at the rate of π cc/s. The rate at which its surface area increases when the radius is 1 cm is

(a) 2π sq. cm/s (b) π sq. cm/s

(c) $\dfrac{3\pi}{2}$ sq. cm/s (d) $\dfrac{\pi}{2}$ sq. cm/s

37. If $P=\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(a) 11 (b) 5 (c) 0 (d) 4

38. If $I_1 = \int_0^{\pi/4} \sin^2 x dx$ and $I_2 = \int_0^{\pi/4} \cos^2 x dx$, then

(a) $I_1 = I_2$ (b) $I_1 < I_2$ (c) $I_1 > I_2$ (d) $I_2 = I_1 + \pi/4$

39. If $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is to be square root of the two rowed unit matrix, then α, β and γ should satisfy the relation

(a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 - \beta\gamma = 0$

(c) $1 - \alpha^2 + \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

40. If $\sqrt{y} = \cos^{-1} x$, then it satisfies the differential equation $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} = c$, where c is equal to

(a) 0 (b) 3 (c) 1 (d) 2

41. If $P=\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, then P^5 equals

(a) P (b) $2P$ (c) $-P$ (d) $-2P$

42. If m and n denote respectively the order and degree of a differential equation

$$\left[a+\left(\frac{dy}{dx}\right)^6\right]^{7/5} = b\frac{d^2y}{dx^2}$$

then the value of (m, n) will be

(a) (1, 7) (b) (1, 6) (c) (2, 5) (d) (2, 6)

43. If α, β, γ are cube roots of unity then the value of

$$\begin{vmatrix} e^{\alpha} & e^{2\alpha} & (e^{3\alpha}-1) \\ e^{\beta} & e^{2\beta} & (e^{3\beta}-1) \\ e^{\gamma} & e^{2\gamma} & (e^{3\gamma}-1) \end{vmatrix} =$$

(a) -2 (b) -1 (c) 0 (d) 1

44. The general solution of differential equation $\dfrac{d^2y}{dx^2} = e^{2x} + e^{-x}$ is

(a) $4e^{2x} + e^{-x} + c_1x + c_2$ (b) $\dfrac{1}{4}e^{2x} - e^{-x} + c$

(c) $\dfrac{1}{4}e^{2x} + e^{-x} + c_1x + c_2$ (d) $\dfrac{1}{4}e^{2x} - e^{-x} + c_1x + c_2$

45. Solution of the differential equation $xdy - ydx = 0$ represents a

(a) parabola (b) circle

(c) hyperbola (d) straight line

46. Five dice are tossed. What is the probability that the five numbers shown will be different?

(a) $\dfrac{5}{54}$ (b) $\dfrac{5}{18}$ (c) $\dfrac{5}{27}$ (d) $\dfrac{5}{81}$

47. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then

(a) $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ (b) $\vec{a}^2 = \vec{b}^2 + \vec{c}^2$

(c) $\vec{a}+\vec{b}=\vec{c}$ (d) None of these

48. If $\int \dfrac{dx}{\cos 2x + 3\sin^2 x} = \dfrac{1}{a}\tan^{-1}(b\tan x) + c$ then, $ab =$

(a) 0 (b) 1 (c) 2 (d) 3

49. $f(x) = x + |x|$ is continuous for

(a) $x \in (-\infty, \infty)$ (b) $x \in (-\infty, \infty) - \{0\}$

(c) only $x > 0$ (d) no value of x

50. Let P be the set of all non-singular matrices of order 3 over R and Q be the set of all orthogonal matrices of order 3 over R. Then

(a) P is proper subset of Q

(b) Q is proper subset of P

(c) Neither P is proper subset of Q nor Q is proper subset of P

(d) $P \cap Q = \phi$, the void set

CATEGORY-II (Q. 51 to Q. 65)

Only one answer is correct. Correct answer will fetch full marks 2. Incorrect answer or any combination of more than one answer will fetch –½ marks. No answer will fetch 0 marks.

51. Let the co-efficients of powers of x in the 2nd, 3rd and 4th terms in the expansion of $(1 + x)^n$, where n is a positive integer, be in arithmetic progression. The sum of the co-efficients of odd powers of x in the expansion is

(a) 32 (b) 64 (c) 128 (d) 256

52. If $\begin{bmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} \end{bmatrix}^k = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Then the least value of k equals $(k \neq 0)$

(a) 1 (b) 2 (c) –1 (d) 3

53. If $\int_0^y e^{-t^2} dt + \int_0^{x^2} \sin^2 t\, dt = 0$, then $\frac{dy}{dx}$ at $x = y = 1$ is

(a) $\sin^2 1$ (b) $-e \sin^2 1$

(c) $-2e \sin^2 1$ (d) none of these

54. If the roots of $ax^2 + bx + c = 0$ are of the form $\frac{m}{m-1}$ and $\frac{m+1}{m}$, then the value of $(a + b + c)^2$ is

(a) $b^2 - 2ac$ (b) $2b^2 - ac$

(c) $b^2 - 4ac$ (d) $2(b^2 - 2ac)$

55. $3^{\frac{1}{9}} \cdot 9^{\frac{1}{27}} \cdot 27^{\frac{1}{81}} \cdot 81^{\frac{1}{243}} \ldots \infty =$

(a) $\sqrt{3}$ (b) 1 (c) $\frac{1}{3}$ (d) none of these

56. The system of linear equations $x_1 + 2x_2 + x_3 = 3$, $2x_1 + 3x_2 + x_3 = 3$ and $3x_1 + 5x_2 + 2x_3 = 1$, has

(a) infinite number of solutions

(b) exactly three solutions

(c) a unique solution (d) no solution

57. Let $f(x) = \begin{cases} x^2 \left|\cos\frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in R$ then f is

(a) differentiable at both $x = 0$ and $x = 2$

(b) differentiable at $x = 0$ but not differentiable at $x = 2$

(c) not differentiable at $x = 0$, but differentiable at $x = 2$

(d) differentiable at neither $x = 0$ nor $x = 2$.

58. An open cylindrical can has to be made with 100 m^2 of tin. If its volume is maximum, the ratio of its base radius and the height is

(a) 1 : 1 (b) 2 : 1 (c) 1 : 3 (d) $\sqrt{2}:1$

59. The value of $1 + \frac{1}{3^2} + \frac{1\cdot 4}{1\cdot 2}\frac{1}{3^4} + \frac{1\cdot 4\cdot 7}{1\cdot 2\cdot 3}\frac{1}{3^6} + \ldots$ is

(a) $\left(\frac{3}{2}\right)^{\frac{1}{3}}$ (b) $\left(\frac{5}{4}\right)^{\frac{1}{3}}$

(c) $\left(\frac{3}{2}\right)^{\frac{1}{6}}$ (d) none of these

60. The function $f(x) = \frac{\lambda \sin x + 6\cos x}{2\sin x + 3\cos x}$ is monotonic increasing when

(a) $\lambda > 1$ (b) $\lambda > 4$ (c) $\lambda < 1$ (d) $\lambda < 4$

61. The value of $\int_1^e \frac{dx}{6x(\log x)^2 + 7x(\log x) + 2x} =$

(a) $\log_e\left(\frac{15}{2}\right)$ (b) $\log_e\left(\frac{6}{5}\right)$

(c) $\log_e\left(\frac{3}{10}\right)$ (d) $\frac{1}{5}\log_e\left(\frac{8}{3}\right)$

62. A body travels a distance s in t seconds. It starts from rest and end at rest. In the first part of the journey it moves with uniform acceleration a and in the second part with constant retardation r. Then the value of t is given by

(a) $2s\left(\frac{1}{a} + \frac{1}{r}\right)$ (b) $\frac{2s}{\frac{1}{a} + \frac{1}{r}}$

(c) $\sqrt{2s\left(\frac{1}{a} + \frac{1}{r}\right)}$ (d) $\sqrt{2s(a + r)}$

63. Let S_K be the sum of an infinite G.P. series whose first term is K and common ratio is $\frac{K}{K+1}$ $(K > 0)$. Then the value of $\sum_{K=1}^{\infty} \frac{(-1)^K}{S_K}$ is equal to

(a) $\log_e 4$ (b) $\log_e 2 - 1$

(c) $1 - \log_e 2$ (d) $1 - \log_e 4$

64. $\frac{d}{dx}\left(\sin^2 \cot\left(\sqrt{\frac{1-x}{1+x}}\right)\right) =$

(a) $-\frac{1}{2}$ (b) –1 (c) 1 (d) $\frac{1}{2}$

65. Let A and B any two events. Which one of the following statements is always true?

(a) $P(A'/B) = P(A/B)$ (b) $P(A'/B) = P(B'/A)$

(c) $P(A'/B) = 1 - P(A/B)$

(d) $P(A'/B) = 1 - P(A/B')$

CATEGORY-III (Q. 66 TO Q. 75)

One or more answer(s) is (are) correct. Correct answer(s) will fetch full marks 2. Any combination containing one or more incorrect answer will fetch 0 marks. Also no answer will fetch 0 marks. If all correct answers are not marked and also no incorrect answer is marked then score = 2 × number of correct answers marked ÷ actual number of correct answers.

66. If $\int 4(3-2x)^{-2}\left(\frac{3-2x}{3+2x}\right)^{\frac{1}{3}} dx = \frac{3}{\alpha}\left(\frac{3+2x}{3-2x}\right)^{\frac{\beta}{\gamma}} + c$

(β and γ are prime nos.) ,then
(a) α, β, γ are in G.P. (b) α, β, γ are in H.P.
(c) α, β, γ are in A.P. (d) $\alpha = \beta\gamma$

67. The function $f(x) = \begin{cases} |x-3|, & x \ge 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ is
(a) continuous at $x = 1$
(b) differentiable at $x = 1$
(c) continuous at $x = 3$
(d) differentiable at $x = 3$

68. The area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is
(a) $e - 1$ (b) $\int_1^e \ln(e+1-y)\, dy$
(c) $e - \int_0^1 e^x\, dx$ (d) $\int_1^x \ln y\, dy$

69. If A and B are independent events such that $0 < P(A) < 1$, $0 < P(B) < 1$ then
(a) A, B are mutually exclusive
(b) A and B' are independent
(c) A' and B' are independent
(d) $P(A|B) + P(A'|B) = 1$

70. If $A^2 - 3A + 2I = O$, then $A =$
(a) I (b) $2I$ (c) $\begin{bmatrix} 3 & -2 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$

71. $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} =$
(a) $\frac{1-\sin\theta}{\cos\theta}$ (b) $\frac{1+\sin\theta}{\cos\theta}$
(c) $\frac{\cos\theta}{1-\sin\theta}$ (d) $\frac{\cos\theta}{1+\sin\theta}$

72. If $\int \frac{\left(\frac{1}{x}+\frac{1}{x^2}\right)(x-1)dx}{\left(\frac{1}{x^4}+\frac{1}{x^2}\right)\sqrt{(x^4-x^3+x^2)(x^4+x^3+x^2)}}$ $= \sec^{-1}\{f(x)\} + c$ then
(a) Maximum value of $f(x) = -2$
(b) Minimum value of $f(x) = 2$
(c) $f(0)$ is not defined
(d) $f(\pi) < f(e)$

73. If z is a point on the circle $|z-1| = 1$, then $\arg z =$
(a) $\arg|z-1|$ (b) $\frac{1}{2}\arg(z-1)$
(c) $\arg(z^2 - z)$ (d) $\frac{1}{3}\arg(z^2-z)$

74. For which of the following values of m, is the area bounded by the curves $y = x - x^2$ and $y = mx$ equals $9/2$?
(a) -4 (b) -2 (c) 2 (d) 4

75. ${}^{2n}P_n$ is equal to
(a) $(n+1)(n+2) \ldots (2n)$
(b) $2^n [1 \cdot 3 \cdot 5. \ldots (2n-1)]$
(c) $(2) \cdot (6) \cdot (10) \ldots (4n-2)$
(d) $n!\ ({}^{2n}C_n)$

SOLUTIONS

1. **(d) :** $\sin^{-1}\left[\tan\left(-\frac{5\pi}{4}\right)\right] = \sin^{-1}\left(-\tan\left(\frac{5\pi}{4}\right)\right)$

$= -\sin^{-1}\left(\tan\left(\pi+\frac{\pi}{4}\right)\right) = -\sin^{-1}\left(\tan\frac{\pi}{4}\right) = -\sin^{-1}(1) = -\frac{\pi}{2}$

2. **(c)**

3. **(d) :** $f(1) + 2f(2) + 3f(3) + \ldots + nf(n)$
$= n(n+1) f(n)$ for $n \ge 2$ ... (i)
Putting $n = n + 1$, we get
$f(1) + 2f(2) + 3f(3) + \ldots + nf(n) + (n+1) f(n+1)$
$= (n+1)(n+2) f(n+1)$ ...(ii)
(ii) – (i) $\Rightarrow (n+1) f(n+1) = (n+1)(n+2) f(n+1) - n(n+1) f(n)$
$\Rightarrow f(n+1) = (n+2) f(n+1) - n f(n)$
$\Rightarrow (n+1) f(n+1) = n f(n)$
$\Rightarrow 2f(2) = 3f(3) = 4f(4) \ldots\ldots = nf(n)$
From (i),
$f(1) + \underbrace{nf(n)+nf(n)+nf(n)+\ldots+nf(n)}_{(n-1)\text{ times}} = n(n+1) f(n)$
$\Rightarrow 1 + (n-1) n f(n) = n(n+1) f(n) \Rightarrow 2n f(n) = 1$
$\therefore f(n) = \frac{1}{2n} \Rightarrow f(500) = \frac{1}{1000}$

4. **(d)** 5. **(b)** 6. **(d)** 7. **(c)**

8. **(b) :** Since, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$

Again, $A^3 = A \times A^2 = A \times 2A = 2A^2 = 2^2 A$
Similarly, $A^{100} = 2^{99} A$

9. (c) **10. (d)** **11. (d)**

12. (d): (i) $\because IAI^{-1} = (AI)I^{-1} = A(II^{-1}) = AI = A$

$\therefore$ $A = IAI^{-1}$, where I and I^{-1} are invertible matrices

$\Rightarrow$ Relation is reflexive

(ii) Let P and Q be invertible matrices

$\therefore$ P^{-1} and Q^{-1} are also invertible

Now, if $B = PAQ^{-1}$ then $BQ = (PAQ^{-1})Q = PAI = PA$

$\Rightarrow$ $P^{-1}(BQ) = P^{-1}(PA) = IA = A \Rightarrow P^{-1}B(Q^{-1})^{-1} = A$

$\Rightarrow$ Relation is symmetric as P^{-1} and Q^{-1} both are invertible.

(iii) Let $A \sim B$ and $B \sim C$

i.e., $B = PAQ^{-1}$ and $C = XBY^{-1}$

$\therefore$ $C = X(PAQ^{-1})Y^{-1} = (XP)A(Q^{-1}Y^{-1}) = (XP)A(YQ)^{-1}$

$\Rightarrow$ $A \sim C$ $\Rightarrow$ Relation is transitive also.

Hence, relation is equivalence.

13. (b)

14. (a): The given series is

$$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\frac{4}{5}x^5+\text{....... to }\infty$$

$$=\frac{2-1}{2}x^2+\frac{3-1}{3}x^3+\frac{4-1}{4}x^4+\frac{5-1}{5}x^5+\text{....... to }\infty$$

$$= x^2 + x^3 + x^4 + x^5 + \text{.... to } \infty - \left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\text{....}\right)$$

$$=\frac{x^2}{1-x}+x+\log(1-x)=\frac{x^2+x-x^2}{1-x}+\log(1-x)$$

$$=\frac{x}{1-x}+\log_e(1-x)$$

15. (a): $A^2=\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\a&b&-1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

$\Rightarrow$ $A^2 = A^4 = A^6 = A^8 = I_3$

Now, $A^2+2A^4+4A^6$

$$=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}+\begin{bmatrix}4&0&0\\0&4&0\\0&0&4\end{bmatrix}=\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}=7A^8$$

16. (a): $\dfrac{(1-3x+x^2)}{e^x} = (1 - 3x + x^2)e^{-x}$

$$=(1-3x+x^2)\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\text{......... to }\infty\right)$$

Thus the co - efficient of x^4 is

$$\frac{1}{2!}+\frac{3}{3!}+\frac{1}{4!}=\frac{1}{2}+\frac{1}{2}+\frac{1}{24}=1+\frac{1}{24}=\frac{25}{24}$$

17. (b)

18. (d): $\dfrac{1}{1\cdot2}-\dfrac{1}{2\cdot3}+\dfrac{1}{3\cdot4}-\dfrac{1}{4\cdot5}+\text{.....to }\infty$

$$=\frac{2-1}{1\cdot2}-\frac{3-2}{2\cdot3}+\frac{4-3}{3\cdot4}-\frac{5-4}{4\cdot5}+\text{.....to }\infty$$

$$=1-\frac{1}{2}-\frac{1}{2}+\frac{1}{3}+\frac{1}{3}-\frac{1}{4}-\frac{1}{4}+\frac{1}{5}+\text{.....to }\infty$$

$$=1-2\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\text{....}\right)$$

$$=1-2[1-\log_e 2]\left[\because \log_e 2=1-\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}+\text{....to }\infty\right)\right]$$

$$= -1 + 2\log_e 2 = \log_e 4 - \log_e e = \log_e\left(\frac{4}{e}\right)$$

19. (a): $\begin{vmatrix}\sin\alpha & \cos\alpha & \sin(\alpha+\delta)\\ \sin\beta & \cos\beta & \sin(\beta+\delta)\\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta)\end{vmatrix}$

$$=\begin{vmatrix}\sin\alpha & \cos\alpha & \sin\alpha\cos\delta+\cos\alpha\sin\delta\\ \sin\beta & \cos\beta & \sin\beta\cos\delta+\cos\beta\sin\delta\\ \sin\gamma & \cos\gamma & \sin\gamma\cos\delta+\cos\gamma\sin\delta\end{vmatrix}$$

$$=\cos\delta\begin{vmatrix}\sin\alpha & \cos\alpha & \sin\alpha\\ \sin\beta & \cos\beta & \sin\beta\\ \sin\gamma & \cos\gamma & \sin\gamma\end{vmatrix}+\sin\delta\begin{vmatrix}\sin\alpha & \cos\alpha & \cos\alpha\\ \sin\beta & \cos\beta & \cos\beta\\ \sin\gamma & \cos\gamma & \cos\gamma\end{vmatrix}$$

$= 0 + 0$ [$\because C_1 = C_3$ and $C_2 = C_3$ in 1st and 2nd determinant]

20. (a)

21. (d): $1^2 + 2^2 + 3^2 + \text{.........} + n^2$

$$=\frac{n(n+1)(2n+1)}{6}=\frac{n(n+1)\left(n+\frac{1}{2}\right)}{3}>\frac{n\cdot n\cdot n}{3}>\frac{n^3}{3}$$

$$\left[\because\ n+1>n \text{ and } n+\frac{1}{2}>n\right]$$

22. (d): We have, $a * b = a^b$

$\therefore$ $(2 * 3) * 2 = (2^3) * 2 = 8 * 2 = 8^2 = 64$

23. (b): $\dfrac{2}{3!}+\dfrac{4}{5!}+\dfrac{6}{7!}+\text{......... to }\infty$

$$=\frac{3-1}{3!}+\frac{5-1}{5!}+\frac{7-1}{7!}+\text{....... to }\infty$$

$$=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}-\frac{1}{7!}+\text{........ to }\infty = e^{-1}$$

24. (d): $P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B)]$ [$\because$ A and B are mutually exclusive]

$= 1 - (0.25 + 0.40) = 1 - 0.65 = 0.35$

25. (c) : $P(A)=\dfrac{{}^1C_1\times{}^4C_1}{{}^5C_2}=\dfrac{4}{10}=\dfrac{2}{5}$

26. (b)

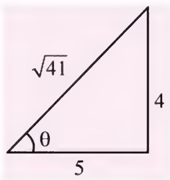
27. (d) : $[5\ 6\ 2]_{1\times 3}\cdot A^T = [4\ 8\ 1\ 7\ 8]_{1\times 5}$
$(1 \times 3) \times (p \times q)$ gives 1×5
$\therefore\quad p = 3$ and $q = 5$
Thus A^T is of order 3×5
$\Rightarrow\quad A$ is of order 5×3

28. (d) : $\dfrac{\pi}{2}-\sin^{-1}x<\sin^{-1}x\Rightarrow \sin^{-1}x>\dfrac{\pi}{4}$

But, $\dfrac{-\pi}{2}\le\sin^{-1}x\le\dfrac{\pi}{2}\Rightarrow\dfrac{\pi}{4}<\sin^{-1}x\le\dfrac{\pi}{2}\Rightarrow x\in\left(\dfrac{1}{\sqrt{2}},1\right]$

29. (b) : Let $P(n) = n^3 + 2n$
Now, $P(1) = 1^3 + 2\cdot 1 = 3$ is divisible by 3
$P(2) = 2^3 + 2\cdot 2 = 12$ is divisible by 3
$P(3) = 3^3 + 2\cdot 3 = 33$ is divisible by 3
In this way we can say $n^3 + 2n$ is always divisible by 3 for $n\in I$.

30. (b) : $\cot^{-1}9+\operatorname{cosec}^{-1}\dfrac{\sqrt{41}}{4}$



$=\cot^{-1}9+\tan^{-1}\dfrac{4}{5}=\tan^{-1}\dfrac{1}{9}+\tan^{-1}\dfrac{4}{5}$

$=\tan^{-1}\left(\dfrac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}\times\frac{4}{5}}\right)=\tan^{-1}\dfrac{41}{41}=\tan^{-1}1=\dfrac{\pi}{4}$

31. (b) : $\displaystyle\int_0^4|x-1|\,dx=\int_0^1(1-x)dx+\int_1^4(x-1)dx$

$=\left[x-\dfrac{x^2}{2}\right]_0^1+\left[\dfrac{x^2}{2}-x\right]_1^4=\left(1-\dfrac{1}{2}\right)+(8-4)-\left(\dfrac{1}{2}-1\right)$

$=\dfrac{1}{2}+4+\dfrac{1}{2}=5.$

32. (d) **33. (d)**

34. (c) : Given that $P(A^c)=\dfrac{2}{3}\Rightarrow P(A)=\dfrac{1}{3}$

and $P(B^c)=\dfrac{2}{7}\Rightarrow P(B)=\dfrac{5}{7}$

Now $P(A\cap B)=P(A)\cdot P(B)=\dfrac{1}{3}\times\dfrac{5}{7}=\dfrac{5}{21}$

35. (d)

36. (a) : Let the volume of the sphere be V.

$\therefore\quad V=\dfrac{4}{3}\pi r^3\quad\Rightarrow\quad\dfrac{dV}{dt}=\dfrac{4}{3}\pi\cdot 3r^2\dfrac{dr}{dt}$

Given that $\dfrac{dV}{dt}=\pi$ cc / sec

$\therefore\quad \pi=\dfrac{4}{3}\pi\cdot 3r^2\dfrac{dr}{dt}\quad\Rightarrow\quad\dfrac{dr}{dt}=\dfrac{1}{4r^2}$ .... (i)

Let S be the surface area of the sphere.

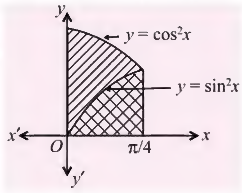
$\therefore\quad S=4\pi r^2\Rightarrow\dfrac{dS}{dt}=8\pi r\dfrac{dr}{dt}=\dfrac{8\pi r}{4r^2}=\dfrac{2\pi}{r}$ (by (i))

or $\left(\dfrac{dS}{dt}\right)_{r=1}=2\pi$ sq. cm / sec $[\because r = 1\text{ cm}]$

37. (a) : Given $P=\begin{bmatrix}1&\alpha&3\\1&3&3\\2&4&4\end{bmatrix}$

$|P| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$
We know that $det(adjA) = (det\ A)^{n-1}$
$\therefore\quad 2\alpha - 6 = 4^{3-1} = 16 \Rightarrow 2\alpha = 22 \Rightarrow \alpha = 11$

38. (b) : $I_1=\displaystyle\int_0^{\frac{\pi}{4}}\sin^2 x\,dx$ is the area of curve bounded between $y = \sin^2 x$, x-axis, $x = 0$ and $x = \pi/4$



$I_2=\displaystyle\int_0^{\frac{\pi}{4}}\cos^2 x\,dx$

is the area of curve bounded between $y = \cos^2 x$, x-axis, $x = 0$ and $x = \pi/4$
From figure, area under I_2 is greater than area under I_1.

39. (b) : **40. (d)**

41. (a) : $P^2=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}$

$=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}=P$

$\therefore\quad P^4 = P^2 = P\quad\Rightarrow\quad P^5 = P^4\cdot P = P\cdot P = P^2 = P.$

42. (c)

43. (c) : $\begin{vmatrix}e^\alpha&e^{2\alpha}&(e^{3\alpha}-1)\\e^\beta&e^{2\beta}&(e^{3\beta}-1)\\e^\gamma&e^{2\gamma}&(e^{3\gamma}-1)\end{vmatrix}$

$=e^{\alpha+\beta+\gamma}\begin{vmatrix}1&e^\alpha&e^{2\alpha}-e^{-\alpha}\\1&e^\beta&e^{2\beta}-e^{-\beta}\\1&e^\gamma&e^{2\gamma}-e^{-\gamma}\end{vmatrix}=\begin{vmatrix}1&e^\alpha&(e^{2\alpha}-e^{-\alpha})\\1&e^\beta&(e^{2\beta}-e^{-\beta})\\1&e^\gamma&(e^{2\gamma}-e^{-\gamma})\end{vmatrix}$

$[\because \alpha,\beta$ and γ are cube roots of unity $\therefore\ \alpha+\beta+\gamma=0]$

$$=\begin{vmatrix} 0 & e^{\alpha}-e^{\beta} & (e^{\alpha}-e^{\beta})(e^{\alpha}+e^{\beta}+e^{-\alpha}e^{-\beta}) \\ 0 & e^{\beta}-e^{\gamma} & (e^{\beta}-e^{\gamma})(e^{\beta}+e^{\gamma}+e^{-\beta}e^{-\gamma}) \\ 1 & e^{\gamma} & e^{2\gamma}-e^{-\gamma} \end{vmatrix}$$

$$=(e^{\alpha}-e^{\beta})(e^{\beta}-e^{\gamma})\begin{vmatrix} 0 & 1 & e^{\alpha}+e^{\beta}+e^{-\alpha}e^{-\beta} \\ 0 & 1 & e^{\beta}+e^{\gamma}+e^{-\beta}e^{-\gamma} \\ 1 & e^{\gamma} & e^{2\gamma}-e^{-\gamma} \end{vmatrix}$$

$$=(e^{\alpha}-e^{\beta})(e^{\beta}-e^{\gamma})(e^{\gamma}-e^{\alpha})\left[1-\frac{1}{e^{\alpha+\beta+\gamma}}\right]$$

$$=(e^{\alpha}-e^{\beta})(e^{\beta}-e^{\gamma})(e^{\gamma}-e^{\alpha})(1-1)=0 \ [\because \alpha+\beta+\gamma=0]$$

44. (c) : $\dfrac{d^2y}{dx^2}=e^{2x}+e^{-x} \Rightarrow \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=e^{2x}+e^{-x}$

$\Rightarrow \int d\left(\dfrac{dy}{dx}\right)=\int(e^{2x}+e^{-x})dx \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2}e^{2x}-e^{-x}+c_1$

$\Rightarrow \int dy=\dfrac{1}{2}\int e^{2x}dx-\int e^{-x}dx+c_1\int dx$

$\Rightarrow y = \dfrac{1}{4}e^{2x} + e^{-x} + c_1x + c_2$

45. (d) : $xdy - ydx = 0 \Rightarrow \dfrac{dx}{x}=\dfrac{dy}{y}$

On integrating, we get $\log y = \log x + \log c$

$\Rightarrow y = cx$, which is a straight line.

46. (a) : Total number of cases $= 6^5$

Number of favourable cases $= 6! = 720$

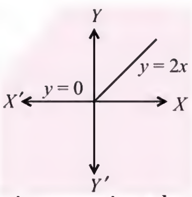
$\therefore$ Required probability $=\dfrac{720}{6^5}=\dfrac{5}{54}$

47. (a) : The position vector of the centroid of any triangle whose position vectors of the vertices are $\vec{a},\vec{b},\vec{c}$, is $\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}$. Since, the triangle is equilateral, therefore the orthocentre coincides with the centroid and hence $\dfrac{\vec{a}+\vec{b}+\vec{c}}{3}=\vec{0} \Rightarrow \vec{a}+\vec{b}+\vec{c}=\vec{0}$

[$\because$ Orthocentre is at origin (given)]

48. (c)

49. (a) : $f(x) = x + |x| = \begin{cases} 2x, & x \ge 0 \\ 0, & x < 0 \end{cases}$



From the graph we see that there is no discontinuity in curve, so $f(x)$ is continuous for $(-\infty, \infty)$

50. (b) : $\because$ Every orthogonal matrix is non-singular but every non-singular matrix may or may not be orthogonal.

$\therefore$ Q is proper subset of P.

51. (b) : Given that ${}^nC_1, {}^nC_2, {}^nC_3 \in$ A.P.

$\therefore \ 2\,{}^nC_2={}^nC_1+{}^nC_3 \Rightarrow 2\cdot\dfrac{n(n-1)}{2!}=n+\dfrac{n(n-1)(n-2)}{3!}$

$\Rightarrow n^2 - 9n + 14 = 0 \Rightarrow n = 7$ $\quad [\because n \ne 2]$

$\therefore$ Sum of the co-efficients of odd powers of x in the expansion of $(1 + x)^n$ is $2^{n-1} = 2^{7-1} = 2^6 = 64$

52. (d) **53. (c)** **54. (c)**

55. (d) **56. (d)** **57. (b)**

58. (a) : Let the radius of the base and height of the cylinder are r and h respectively.

$\therefore \ 2\pi rh + \pi r^2 = 100 \Rightarrow 2\pi rh = 100 - \pi r^2$

$\Rightarrow h=\dfrac{100}{2\pi r}-\dfrac{\pi r^2}{2\pi r}=\dfrac{50}{\pi r}-\dfrac{r}{2}$

Now $V=\pi r^2h=\pi r^2\left(\dfrac{50}{\pi r}-\dfrac{r}{2}\right) \Rightarrow \dfrac{dV}{dr}=50-\dfrac{3\pi r^2}{2}$

$\therefore \ \dfrac{dV}{dr}=0$ gives $r=\dfrac{10}{\sqrt{3\pi}}$

Now $\dfrac{d^2V}{dr^2}=-3\pi r \quad \therefore \left(\dfrac{d^2V}{dr^2}\right)_{r=\frac{10}{\sqrt{3\pi}}}<0$

Thus V is maximum when $r=\dfrac{10}{\sqrt{3\pi}}$

$\therefore \ h=\dfrac{50}{\pi\cdot\frac{10}{\sqrt{3\pi}}}-\dfrac{10}{2\sqrt{3\pi}}=\dfrac{5\sqrt{3}}{\sqrt{\pi}}-\dfrac{5}{\sqrt{3\pi}}=\dfrac{10}{\sqrt{3\pi}}$

So, $r : h = 1 : 1$

59. (a)

60. (b) : $f(x)=\dfrac{\lambda\sin x+6\cos x}{2\sin x+3\cos x}$

$f'(x)=\dfrac{3\lambda-12}{(2\sin x+3\cos x)^2}$

$\therefore$ $f(x)$ is monotonic increasing function, hence $f'(x) > 0 \Rightarrow 3\lambda - 12 > 0$ or $\lambda > 4$.

61. (b) : Let $I=\displaystyle\int_1^e\frac{dx}{6x(\log x)^2+7x(\log x)+2x}$

$=\displaystyle\int_1^e\frac{dx}{x\left[6(\log x)^2+7(\log x)+2\right]}=\int_1^e\frac{d(\log x)}{(3\log x+2)(2\log x+1)}$

Let $\dfrac{1}{(3\log x+2)(2\log x+1)}=\dfrac{A}{3\log x+2}+\dfrac{B}{2\log x+1}$

$1 = A(2\log x + 1) + B(3\log x + 2)$

Put $\log x = -\dfrac{1}{2} \Rightarrow 1=B\left(2-\dfrac{3}{2}\right)=\dfrac{B}{2} \Rightarrow B = 2$

Put $\log x = -\frac{2}{3} \Rightarrow 1 = A\left(1-\frac{4}{3}\right) \Rightarrow A = -3$

$$\therefore \quad I = -3\int_1^e \frac{d(\log x)}{3\log x+2} + 2\int_1^e \frac{d(\log x)}{2\log x+1}$$

$$= -3\cdot\frac{1}{3}[\log(3\log x+2)]_1^e + 2\cdot\frac{1}{2}[\log(2\log x+1)]_1^e = \log\left(\frac{6}{5}\right)$$

62. (c) **63. (d)**

64. (d) : $\frac{d}{dx}\left[\sin^2 \cot^{-1}\sqrt{\frac{1-x}{1+x}}\right]$

$$= \frac{d}{dx}\left[\sin^2 \cot^{-1} \tan\frac{\theta}{2}\right] \quad [\text{Put } x = \cos\theta]$$

$$= \frac{d}{dx}\left[\sin^2\left(\frac{\pi}{2}-\frac{\theta}{2}\right)\right] = \frac{d}{dx}\left[\cos^2\frac{\theta}{2}\right] = \frac{d}{d\theta}\left[\cos^2\frac{\theta}{2}\right]\cdot\frac{d\theta}{dx}$$

$$= -2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\times\frac{1}{2}\times\left(-\frac{1}{\sin\theta}\right) = \frac{1}{2}$$

65. (c) : $P(A'/B) = \frac{P(A'\cap B)}{P(B)} = \frac{P(B)-P(A\cap B)}{P(B)}$

$$= 1-\frac{P(A\cap B)}{P(B)} = 1 - P(A/B)$$

66. (d) : Let $I = \int 4(3-2x)^{-2}\left(\frac{3-2x}{3+2x}\right)^{1/3} dx$

$$= \int \frac{4}{(3-2x)^2}\cdot\left(\frac{3+2x}{3-2x}\right)^{-1/3} dx$$

Put $\frac{3+2x}{3-2x} = z^3 \Rightarrow \frac{4dx}{(3-2x)^2} = z^2 dz$

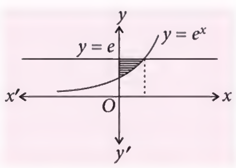
$$\therefore \quad I = \int z^2\times\frac{1}{z}dz = \int z\,dz = \frac{1}{2}\left(\frac{3+2x}{3-2x}\right)^{2/3} + c$$

$$= \frac{3}{6}\left(\frac{3+2x}{3-2x}\right)^{2/3} + c$$

$\therefore \quad \alpha = 6, \beta = 2$ and $\gamma = 3$

67. (a, b, c)

68. (b, c) : The given curves are $y = e^x$, $x = 0$ and $y = e$



Area, $A = \int_1^e x\,dy = \int_1^e \log y\,dy$

$$= \int_1^e \log(e+1-y)\,dy \quad \left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$$

Further, $A = e\times 1 - \int_0^1 e^x dx = e - \int_0^1 e^x dx$

69. (b,c,d) **70. (a,b,c,d)** **71. (b, c)**

72. (a,b,c): $\int \frac{\left(\frac{1}{x}+\frac{1}{x^2}\right)(x-1)dx}{\left(\frac{1}{x^4}+\frac{1}{x^2}\right)\sqrt{(x^4-x^3+x^2)(x^4+x^3+x^2)}}$

$$= \int \frac{\frac{(1+x)(x-1)}{x^2}dx}{\left(\frac{1}{x^4}+\frac{1}{x^2}\right)(x^8+x^6+x^4)^{\frac{1}{2}}} = \int \frac{\frac{x^2-1}{x^2}dx}{\frac{1}{x^3}\left(x+\frac{1}{x}\right)\cdot x^3\sqrt{x^2+1+\frac{1}{x^2}}}$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-1}} = \int \frac{d\left(x+\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-1}}$$

$$= \sec^{-1}\left(x+\frac{1}{x}\right)+c$$

$\therefore \quad f(x) = x + \frac{1}{x} \Rightarrow f'(x) = 1 - \frac{1}{x^2}$

Now $f'(x) = 0$ gives $1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$

$f''(x) = \frac{2}{x^3} \Rightarrow f''(1) = 2 > 0$ and $f''(-1) = -2 < 0$

$\therefore \quad f_{max} = f(-1) = -2$ and $f_{min} = f(1) = 2$

Also, $f(0)$ is undefined.

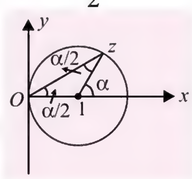
$\therefore \quad f(\pi) = \pi + \frac{1}{\pi} \Rightarrow f(e) = e + \frac{1}{e}$, clearly $f(\pi) > f(e)$.

73. (b, d) : If $\arg(z-1) = \alpha$, then $\arg z = \frac{\alpha}{2}$

$\arg(z^2 - z) = \arg z(z-1)$

$$= \arg z + \arg(z-1) = \frac{3\alpha}{2}$$

$$\therefore \arg z = \frac{1}{2}\arg(z-1) = \frac{1}{3}\arg(z^2-z)$$



74. (b, d)

75. (a, b, c, d) : We have ${}^{2n}P_n = \frac{(2n)!}{(2n-n)!} = \frac{(2n)!}{n!}$ ...(i)

$$= \frac{(2n)(2n-1)(2n-2)(2n-3)...4\cdot 3\cdot 2\cdot 1}{n!}$$

$$= \frac{2^n[n(n-1)(n-2)...2\cdot 1][1\cdot 3\cdot 5...(2n-1)]}{n!}$$

$= 2^n\,[1\cdot 3\cdot 5 \ldots (2n-1)] = 2\cdot 6\cdot 10\ldots(4n-6)(4n-2)$

Also, from (i), ${}^{2n}P_n = (2n)(2n-1)\ldots(n+2)(n+1)$

Lastly, $n!\,({}^{2n}C_n) = (n!)\frac{(2n)!}{n!\,n!} = \frac{(2n)!}{n!} = {}^{2n}P_n$

MATHS MUSING

SOLUTION SET-182

1. **(d) :** Let $z = \lim_{n\to\infty} (z_1 z_2 \ldots z_n)$

$$\theta = \arg(z) = \arg(\lim_{n\to\infty}(z_1 z_2 z_3 \ldots z_n))$$

$$\Rightarrow \theta = \arg\left(\lim_{n\to\infty} \text{cis}\pi\left(\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\ldots+\frac{1}{n(n+1)(n+2)}\right)\right)$$

Let $S_n = \frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\ldots+\frac{1}{n(n+1)(n+2)}$

$S_n = c - \left(\frac{1}{(n+1)(n+2)2}\right)$ (c is constant)

Put $n = 1 \Rightarrow S_1 = c - \frac{1}{2\cdot3\cdot2}$

$$\Rightarrow \frac{1}{1\cdot2\cdot3} = c - \frac{1}{12} \Rightarrow c = \frac{1}{6}+\frac{1}{12} = \frac{2+1}{12} = \frac{1}{4}$$

$$\lim_{n\to\infty} S_n = \frac{1}{4} - \lim_{n\to\infty}\frac{1}{2(n+1)(n+2)} = \frac{1}{4}$$

$$\theta = \arg\left(\text{cis}\frac{\pi}{4}\right) \quad \therefore \theta = \frac{\pi}{4}$$

2. **(b) :** $(1-3x+6x^2-10x^3+\ldots\infty)^{-n} = ((1+x)^{-3})^{-n} = (1+x)^{3n}$

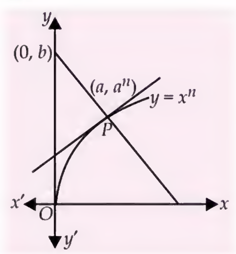
Now, for the coefficient of x^n

consider $T_{r+1} = {}^{3n}C_r (x)^r = {}^{3n}C_r x^r$

Taking $r = n$ for coefficient of x^n

$\therefore$ coefficient of $x^n = {}^{3n}C_n = \frac{3n!}{n!\ 2n!}$

3. **(c):** $y = x^n \Rightarrow \frac{dy}{dx} = nx^{n-1}$



Slope of the normal at (a, a^n)

$$= -\frac{1}{na^{n-1}}$$

Equation of the normal is

$$y - a^n = \frac{-1}{na^{n-1}}(x-a)$$

Putting $x = 0$, we get

$$y = a^n + \frac{1}{na^{n-2}} \Rightarrow b = a^n + \frac{1}{na^{n-2}}$$

Now, $\lim_{a\to0} b = \frac{1}{2} \Rightarrow n = 2$

4. **(c):** $e^{i\theta} = \cos\theta + i\sin\theta$

$\Rightarrow$ The given series is the real part of

$$\sum_{r=0}^{n}\binom{n}{r} a^{n-r}\cdot b^r e^{i[rA-(n-r)B]} = \sum_{r=0}^{n}\binom{n}{r}(ae^{-iB})^{n-r}\cdot(be^{iA})^r$$

$= (ae^{-iB} + be^{iA})^n$

$= (a(\cos B - i\sin B) + b(\cos A + i\sin A))^n$

$\therefore$ Required result $= (a\cos B + b\cos A)^n = c^n$

5. **(c):** We have, $I = \int_0^1 \frac{8\log(1+x)}{1+x^2}dx$

Let $J = \int_0^1 \frac{\log(1+x)}{1+x^2}dx$

Let $x = \tan\theta \Rightarrow J = \int_0^{\pi/4}\log(1+\tan\theta)d\theta$

Now, $J = \int_0^{\pi/4}\log\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$

On adding,

$$2J = \int_0^{\pi/4}\left[\log(1+\tan\theta)+\log\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right]d\theta$$

$$= \int_0^{\pi/4}\log\left\{(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)\right\}d\theta$$

$$\Rightarrow 2J = \int_0^{\pi/4}(\log2)d\theta = \frac{\pi}{4}\log2 \Rightarrow 8J = 4\frac{\pi}{4}\log2$$

$$\Rightarrow I = 8J = \pi\log2$$

6. **(d) :** $2\cot^2\theta + 2\sqrt{3}\cot\theta + 4\text{cosec}\theta + 8 = 0$

$$\Rightarrow (\cot\theta+\sqrt{3})^2 + (\text{cosec}\theta+2)^2 = 0 \Rightarrow \cot\theta = -\sqrt{3},$$

$$\text{cosec}\theta = -2 \Rightarrow \theta = 2n\pi - \frac{\pi}{6}$$

(7 - 8) :

7. **(d)** 8. **(d)**

$$\Delta = \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

MPP CLASS XII ANSWER KEY

1.	(a)	2.	(a)	3.	(b)	4.	(a)	5.	(a)
6.	(c)	7.	(a, d)	8.	(a, b, c, d)			9.	(c, d)
10.	(c, d)	11.	(a)	12.	(a, c)	13.	(a)	14.	(a)
15.	(d)	16.	(c)	17.	(0)	18.	(8)	19.	(2)
20.	(3)								

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = D^2 \text{ (say)}$$

$$\text{Now } D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = (1-\alpha)(\alpha-\beta)(\beta-1)$$

$= (\beta - \alpha)[\alpha\beta - \alpha - \beta + 1]$

$= (\beta - \alpha)\left(\frac{c}{a} + \frac{b}{a} + 1\right) = \frac{(\beta-\alpha)}{a}(a+b+c)$

$\therefore \quad \Delta = D^2 = \frac{(\beta-\alpha)^2}{a^2}(a+b+c)^2$

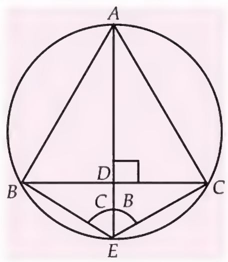
$= \frac{1}{a^2}(a+b+c)^2 \cdot \left[\frac{b^2}{a^2} - 4\frac{c}{a}\right] = \frac{1}{a^4}(a+b+c)^2(b^2-4ac)$

If one root is $1+\sqrt{2}$ and since coefficients are rational, the other root is $1 - \sqrt{2}$.

Hence the equation is $x^2 - 2x - 1 = 0$.

Then the value of $\Delta = (1 - 2 - 1)^2 (4 - 4(1)(-1)) = 32$.

9. (2) : Let AD be the perpendicular from A on BC. When AD is produced, it meets the circumscribing circle at E. From question, $DE = \alpha$.



Since angles in the same segment are equal.

$\therefore \quad \angle AEB = \angle ACB = \angle C$

and $\angle AEC = \angle ABC = \angle B$.

From right-angled triangle BDE, we get

$\tan C = \frac{BD}{DE}$ ... (i)

From the right-angled triangle CDE, we get

$\tan B = \frac{CD}{DE}$ ... (ii)

Adding (i) and (ii), we get

$\tan B + \tan C = \frac{BD+CD}{DE} = \frac{BC}{DE} = \frac{a}{\alpha}$ ... (iii)

Similarly, $\tan C + \tan A = \frac{b}{\beta}$ ... (iv)

and $\tan A + \tan B = \frac{c}{\gamma}$ ... (v)

$\therefore \quad \frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 2(\tan A + \tan B + \tan C)$

10. (b) : (P) $\to$ 2, (Q) $\to$ 1, (R) $\to$ 4, (S) $\to$ 3

(P) We have, $x^2 + 2x\sin(xy) + 1 = 0$

$\Rightarrow \quad [x + \sin(xy)]^2 + 1 - \sin^2 xy = 0$

$\Rightarrow \quad x + \sin(xy) = 0 \,\&\, \sin^2 xy = 1$

If $\sin^2(xy) = 1 \Rightarrow \sin(xy) = 1$ or -1

If $\sin(xy) = 1 \Rightarrow x = -1 \Rightarrow \sin(-y) = 1 \Rightarrow y = 3\pi/2$

If $\sin(xy) = -1 \Rightarrow x = 1 \Rightarrow \sin y = -1 \Rightarrow y = \frac{3\pi}{2}$

Thus 2 ordered pairs.

(Q) $5\sin\theta + 3(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$

$\qquad = (5 + 3\cos\alpha)\sin\theta - 3\sin\alpha\cos\theta$

Max. values $= \sqrt{(5+3\cos\alpha)^2 + 9\sin^2\alpha} = \sqrt{34 + 30\cos\alpha}$

$\therefore \quad 34 + 30\cos\alpha = 49$

$\Rightarrow \quad \cos\alpha = 1/2$

(R) $\cos x = |\sin x - \cos x|$

(i) If $\sin x < \cos x \Rightarrow x \in \left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

$\sin x = 0 \quad \Rightarrow \quad x = 0, 2\pi$

(ii) If $\sin x \ge \cos x \quad \Rightarrow x \in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

$\tan x = 2 \quad \Rightarrow \quad x = \tan^{-1}2$

(S) $\sin\left(\frac{1}{3}\cos^{-1}x\right) = 1 \Rightarrow \cos^{-1}x = \frac{3\pi}{2}$

$\Rightarrow$ There exist no solution. $[\because x \in [0, \pi]]$

EXAM CORNER 2018

Exam	Date
VITEEE	4th to 15th April
JEE Main	8th April (Offline), 15th & 16th April (Online)
SRMJEEE	16th to 30th April
Karnataka CET	18th & 19th April
WBJEE	22nd April
Kerala PET	23rd & 24th April
MHT CET	10th May
COMEDK (Engg.)	13th May
AMU (Engg.)	13th May (Revised)
BITSAT	16th to 31st May
JEE Advanced	20th May

MPP MONTHLY Practice Problems

Class XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ..., is

(a) $\frac{7}{9}(99-10^{-20})$ (b) $\frac{7}{81}(179+10^{-20})$

(c) $\frac{7}{9}(99+10^{-20})$ (d) $\frac{7}{81}(179-10^{-20})$

2. The value of

$\tan\alpha + 2\tan(2\alpha) + 4\tan(4\alpha) + \ldots + 2^{n-1}\tan(2^{n-1}\alpha) + 2^n\cot(2^n\alpha)$ is

(a) $\cot(2^n\alpha)$ (b) $2^n\tan(2^n\alpha)$

(c) 0 (d) $\cot\alpha$

3. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m=1}^{15} \text{Im}(z^{2m-1})$ at $\theta = 2°$ is

(a) $\frac{1}{\sin 2°}$ (b) $\frac{1}{3\sin 2°}$

(c) $\frac{1}{2\sin 2°}$ (d) $\frac{1}{4\sin 2°}$

4. Consider three points $P \equiv (-\sin(\beta-\alpha), -\cos\beta)$, $Q \equiv (\cos(\beta-\alpha), \sin\beta)$ and $R \equiv (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$, where $0 < \alpha, \beta, \theta < \pi/4$. Then,

(a) P lies on the line segment RQ

(b) Q lies on the line segment PR

(c) R lies on the line segment QP

(d) P, Q, R are non-collinear

5. The coefficient of x^{-5} in the binomial expansion of

$$\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$$, where $x \neq 0, 1$, is

(a) 1 (b) −4 (c) −1 (d) 4

6. The equation of the bisector of the acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ is

(a) $99x - 27y - 81 = 0$ (b) $11x - 3y + 9 = 0$

(c) $21x + 77y - 101 = 0$ (d) $21x + 77y + 101 = 0$

One or More Than One Option(s) Correct Type

7. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 10}. The probability that minimum of the chosen number is 3 or their maximum is 7, cannot exceed

(a) $\frac{11}{30}$ (b) $\frac{11}{40}$ (c) $\frac{11}{50}$ (d) $\frac{11}{60}$

8. If $bx + cy = a$, where a, b, c are of the same sign, be a line such that the area enclosed by the line and the axes of reference is $\frac{1}{8}$ sq. unit, then

(a) b, a, c are in G.P. (b) $b, 2a, c$ are in G.P.

(c) $b, \frac{a}{2}, c$ are in A.P. (d) $b, -2a, c$ are in G.P.

9. Let a and b respectively be the semi-transverse and semi-conjugate axes of a hyperbola whose eccentricity satisfies the equation $9e^2 - 18e + 5 = 0$. If $S(5, 0)$ is a focus and $5x = 9$ is the corresponding directrix of this hyperbola, then $a^2 - b^2$ is equal to

(a) −7 (b) −5 (c) 5 (d) 7

10. Let $0 \leq \theta \leq \pi/2$ and $x = X\cos\theta + Y\sin\theta$, $y = X\sin\theta - Y\cos\theta$ such that $x^2 + 4xy + y^2 = aX^2 + bY^2$, where a and b are constants. Then,

(a) $a = -1, b = 3$ (b) $\theta = \frac{\pi}{4}$

(c) $a = 3, b = -1$ (d) $\theta = \frac{\pi}{3}$

11. The area of a triangle is 5 sq. units. Two of its vertices are (2, 1) and (3, –2). The third vertex lies on $y = x + 3$. The coordinates of the third vertex can be

(a) (–3/2, 3/2) (b) (3/4, –3/2)
(c) (7/2, 13/2) (d) (–1/4, 11/4)

12. If $c \neq 0$ and the equation $\frac{p}{2x} = \frac{a}{x+c} + \frac{b}{x-c}$ has two equal roots, then p can be

(a) $\left(\sqrt{a}-\sqrt{b}\right)^2$ (b) $\left(\sqrt{a}+\sqrt{b}\right)^2$
(c) $a + b$ (d) $a - b$

13. The area of a regular polygon of n sides (where, r is inradius, R is circumradius and a is side of the triangle) is

(a) $\frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$ (b) $nr^2\tan\left(\frac{\pi}{n}\right)$
(c) $\frac{na^2}{4}\cot\frac{\pi}{n}$ (d) $nR^2\tan\left(\frac{\pi}{n}\right)$

Comprehension Type

Let $z = a + ib = (a, b)$ be any complex number $\forall\, a, b, \in R$ and $i = \sqrt{-1}$. If $(a, b) \neq (0, 0)$, then $\arg(z) = \tan^{-1}\left(\frac{b}{a}\right)$ where $-\pi < \arg(z) \le \pi$ and

$$\arg(\bar{z}) + \arg(-z) = \begin{cases} \pi, & \text{if } \arg(z) < 0 \\ -\pi, & \text{if } \arg(z) > 0 \end{cases}$$

14. Let z and w be two non zero complex numbers such that $|z| = |w|$ and $\arg(z) + \arg(w) = \pi$, then z equals

(a) w (b) $-w$
(c) $\bar{w}$ (d) $-\bar{w}$

15. The value of

$$\sqrt{\{\arg(z)+\arg(-\bar{z})-2\pi\}\{\arg(-z)+\arg(\bar{z})\}}$$

$\forall\ z = x + iy$, (where $i = \sqrt{-1}$) $x, y > 0$ is

(a) π
(b) $-\pi$
(c) 0
(d) Not defined

Matrix Match Type

16. P is a point inside a triangle ABC. Column II gives $\Delta PBC : \Delta PCA : \Delta PAB$.

Column I		Column II	
P.	P is centroid G	1.	$\sin A : \sin B : \sin C$
Q.	P is incentre I	2.	1 : 1 : 1
R.	P is orthocentre O	3.	$\tan A : \tan B : \tan C$
S.	P is circumcentre S	4.	$\sin 2A : \sin 2B : \sin 2C$

	P	Q	R	S
(a)	3	1	2	4
(b)	3	2	1	4
(c)	1	2	3	4
(d)	2	1	3	4

Integer Answer Type

17. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the square of the other, then $\frac{a+b}{h} + \frac{8h^2}{ab}$ is

18. If $f: R \to R$ is given by

$$f(x) = \begin{cases} -1, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

then $|(fof)(1-\sqrt{3})|$ is equal to

19. The line joining A $(b\cos\alpha, b\sin\alpha)$ and $B(a\cos\beta, a\sin\beta)$ is produced to the point $M(x, y)$, so that $AM : MB = b : a$, then

$x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\left(\frac{\alpha+\beta}{2}\right)$ is

20. Find the value of $\lim_{x \to \frac{\pi}{2}} \frac{\sin x - (\sin x)^{\sin x}}{1 - \sin x + \ln \sin x}$.

◈◈

Keys are published in this issue. Search now! ☺

SELF CHECK

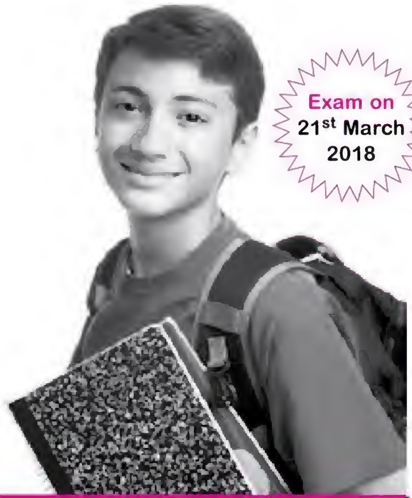
No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

ACE YOUR WAY CBSE XII

Exam on 21st March 2018



PRACTICE PAPER 2018

Time Allowed : 3 hours | Maximum Marks : 100

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) This question paper contains **29** questions.

(iii) Questions **1-4** in Section-A are very short-answer type questions carrying **1** mark each.

(iv) Questions **5-12** in Section-B are short-answer type questions carrying **2** marks each.

(v) Questions **13-23** in Section-C are long-answer-**I** type questions carrying **4** marks each.

(vi) Questions **24-29** in Section-D are long-answer-**II** type questions carrying **6** marks each.

SECTION - A

1. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ not to be transitive.

2. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, write the minor of the element a_{23}.

3. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

4. Write the direction cosines of a line equally inclined to the three coordinate axes.

SECTION - B

5. Find the value of k so that the following function is continuous at $x = 2$.

$$f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x-2)^2} & ; \quad x \neq 2 \\ k & ; \quad x = 2 \end{cases}$$

6. If $f : R \to R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

7. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to x-axis.

8. Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$.

9. The x-coordinate of a point on the line joining the points $P(2, 2, 1)$ and $Q(5, 1, -2)$ is 4. Find its z-coordinate.

10. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate an L.P.P. for finding how many of each should be produced daily to maximize the profit ? It is being given that at least one of each must be produced.

11. Prove that if E and F are independent events, then the events E' and F' are also independent.

12. Find the angle between the following pair of lines $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z+5}{4}$.

SECTION - C

13. Show that the matrix $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 5A + 7I = O$. Hence, find A^{-1}.

OR

Using properties of determinants, show that

$$\begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix} = (x+y+z)(x-z)^2.$$

14. Verify Rolle's theorem for the function $f(x) = x^2 - 4x + 3$ on $[1, 3]$.

15. If $f(x) = 3x^2 + 15x + 5$, then find the approximate value of $f(3.02)$, using differentials.

OR

Find the values of x for which $f(x) = [x(x-2)]^2$ is an increasing function. Also, find the points on the curve where tangent is parallel to x-axis.

16. Evaluate : $\int \frac{5x+3}{\sqrt{x^2+4x+10}} dx$

17. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

18. Find the particular solution of the differential equation $e^x \sqrt{1-y^2}\, dx + \frac{y}{x} dy = 0$ given that $y = 1$ when $x = 0$.

OR

Form the differential equation representing the family of ellipses having foci on x-axis and centre at the origin.

19. Find a unit vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ where $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$.

20. Find the cartesian equation of the plane passing through the points $A(0, 0, 0)$ and $B(3, -1, 2)$ and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$.

21. If a young man rides his motorcycle at 25 km per hour, he had to spend ₹2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km per hour, the petrol cost increases to ₹5 per km and rate of pollution also increases. He has ₹100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this problem as an L.P.P. Solve it graphically to find the distance to be covered with different speeds. What value is indicated in this question?

22. How many times must a man toss a fair coin, so that the probability of having atleast one head is more than 80%?

23. There are two bags, bag I and bag II. Bag I contains 4 white and 3 red balls while another bag II contains 3 white and 7 red balls. One ball is drawn at random from one of the bags and it is found to be white. Find the probability that it was drawn from bag I.

SECTION - D

24. Evaluate : $\int_1^3 (2x^2+3)\, dx$ as limit of sums.

OR

Find : $\int \frac{\sqrt{x^2+1}[\log(x^2+1) - 2\log x]}{x^4} dx$

25. Using elementary transformations, find the inverse of the matrix.

$$\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}$$

OR

Two institutions decided to award their employees for the three values of resourcefulness, competence and determination in the form of prizes at the rate of ₹ x, ₹ y and ₹ z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of ₹ 37,000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of ₹ 47,000. If all the three prizes per person together amount to ₹ 12,000 then using matrix method find the value of x, y and z.
What values are described in the question?

26. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, prove that

$$\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$$

OR

Show that the binary operation * on $A = R - \{-1\}$ defined as $a * b = a + b + ab$ for all $a, b \in A$ is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible.

27. Show that a right circular cylinder which is open at the top and has a given surface area, will have the greatest volume, if its height is equal to the radius of its base.

28. Find the distance between the point (7, 2, 4) and the plane determined by the points $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.

29. If $f(x)$ defined by the following is continuous at $x = 0$, find the value of a, b and c.

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x}, & \text{if } x < 0 \\ c, & \text{if } x = 0 \\ \dfrac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & \text{if } x > 0 \end{cases}$$

SOLUTIONS

1. For transitivity of a relation,
If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$
We have, $R = \{(1, 2), (2, 1)\}$
$(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$
$\therefore$ R is not transitive.

2. Minor of $a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$

3. Here, $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \left[\cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{-\pi}{6}\right)\right)\right]$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{4}$$

4. Here, line is equally inclined to the axes.
$\therefore$ $l = m = n$. We know that, $l^2 + m^2 + n^2 = 1$

So, $l^2 + l^2 + l^2 = 1 \Rightarrow 3l^2 = 1 \Rightarrow l = \pm\frac{1}{\sqrt{3}}$

$\Rightarrow$ Direction cosines are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.

5. $\because$ $f(x)$ is continuous at $x = 2$

$\therefore$ $\lim\limits_{x\to 2} f(x) = f(2) = k$ ...(i)

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^3 + x^2 - 16x + 20}{(x-2)^2}$$

$$= \lim_{x\to 2} \frac{(x-2)^2(x+5)}{(x-2)^2} = \lim_{x\to 2}(x+5) = 7$$

$\therefore$ From (i), we get $k = 7$

6. Since, we have $f(x) = x^2 - 3x + 2$
$\therefore$ $f(f(x)) = f(x^2 - 3x + 2)$
$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2 = x^4 - 6x^3 + 10x^2 - 3x$

7. The given curve is $x^2 + y^2 - 2x - 3 = 0$. ...(i)
Differentiating with respect to x, we get

$$2x + 2y\frac{dy}{dx} - 2 = 0 \Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$$

Since the tangent is parallel to x-axis,

$$\therefore \frac{dy}{dx} = 0 \Rightarrow \frac{1-x}{y} = 0 \Rightarrow x = 1$$

Putting the value of $x = 1$ in (i), we get
$(1)^2 + y^2 - 2(1) - 3 = 0 \Rightarrow y^2 = 4 \Rightarrow y = \pm 2$
$\therefore$ The points are (1, 2) and (1, –2).

8. We have, $\frac{dy}{dx} + \frac{1}{\sqrt{x}}y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

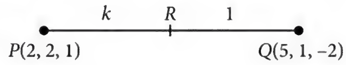
This is a linear differential equation of the form

$$\frac{dy}{dx} + Py = Q, \text{ where } P = \frac{1}{\sqrt{x}}, Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$$

$$\therefore \text{ I.F.} = e^{\int P\,dx} \Rightarrow \text{I.F.} = e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$$

9. Given that $P(2, 2, 1)$ and $Q(5, 1, -2)$
Let the point R on the line PQ, divides the line in the ratio $k : 1$ and x-coordinate of point R on the line is 4.

$$\therefore 4 = \frac{5k+2}{k+1} \Rightarrow k = 2$$



Now, z-coordinate of point R,

$$z = \frac{-2k+1}{k+1} = \frac{-2\times 2+1}{2+1} = -1$$

10. Let x necklaces and y bracelets be manufactured per day to maximize the profit.
$\therefore$ Maximize $Z = 100x + 300y$
Subject to the constraints : $x + y \le 24$,

$$(1)x + \left(\frac{1}{2}\right)y \le 16 \Rightarrow 2x + y \le 32$$

and $x \ge 1, y \ge 1 \Rightarrow x - 1 \ge 0$ and $y - 1 \ge 0$

11. Since, E and F are independent events.
$\therefore$ $P(E \cap F) = P(E)\,P(F)$ ...(i)
Now, $P(E' \cap F') = 1 - P(E \cup F)$
$= 1 - [P(E) + P(F) - P(E \cap F)]$
$= 1 - P(E) - P(F) + P(E)\,P(F)$ [Using (i)]
$= (1 - P(E))\,(1 - P(F)) = P(E')\,P(F')$
Hence, E' and F' are independent events.

12. Standard form of given lines are

$$\frac{x-2}{2} = \frac{y-1}{7} = \frac{z-(-3)}{-3} \text{ and } \frac{x-(-2)}{-1} = \frac{y-4}{2} = \frac{z-(-5)}{4}$$

Since, $\cos\theta = \left|\dfrac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$

where $\vec{b}_1 = 2\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{b}_2 = -\hat{i} + 2\hat{j} + 4\hat{k}$

$$\Rightarrow \cos\theta = \left|\frac{2(-1)+7(2)+(-3)(4)}{\sqrt{(2)^2+(7)^2+(-3)^2}\sqrt{(-1)^2+2^2+4^2}}\right| = 0$$

$\Rightarrow \quad \theta = \pi/2$

13. We have, L.H.S. $= A^2 - 5A + 7I$

$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = \text{R.H.S.}$$

$\Rightarrow \quad A^2 - 5A + 7I = O$

As $|A| \neq 0$, so post-multiplying by A^{-1} on both sides, we get

$A(AA^{-1}) - 5AA^{-1} + 7IA^{-1} = O \Rightarrow AI - 5I + 7A^{-1} = O$

$\Rightarrow \quad A - 5I + 7A^{-1} = O \Rightarrow 7A^{-1} = 5I - A$

$$\Rightarrow \quad 7A^{-1} = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad 7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

OR

$$\text{L.H.S.} = \begin{vmatrix} y+z & x & y \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\begin{vmatrix} 2(x+y+z) & x+y+z & x+y+z \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

Taking $(x + y + z)$ common from R_1, we get

$$(x+y+z)\begin{vmatrix} 2 & 1 & 1 \\ z+x & z & x \\ x+y & y & z \end{vmatrix}$$

Applying $C_1 \to C_1 - 2C_3$, $C_2 \to C_2 - C_3$, we get

$$(x+y+z)\begin{vmatrix} 0 & 0 & 1 \\ z-x & z-x & x \\ x+y-2z & y-z & z \end{vmatrix}$$

$= (x + y + z)\,[(z - x)\,(y - z) - (z - x)\,(x + y - 2z)]$

$= (x + y + z)\,(z - x)^2 = (x + y + z)\,(x - z)^2 = \text{R.H.S.}$

14. We have, $f(x) = x^2 - 4x + 3$

$f(x)$ being a polynomial function

$\therefore \quad f(x)$ is continuous on $[1,3]$ and differentiable on $(1, 3)$.

Also, $f(3) = 3^2 - 4(3) + 3 = 0$

and $f(1) = 1^2 - 4(1) + 3 = 0$. Thus $f(1) = f(3)$

Thus, all the conditions of Rolle's theorem are satisfied, so there exists atleast one point $c \in (1, 3)$ such that $f'(c) = 0$

$f'(x) = 2x - 4 \Rightarrow f'(c) = 2c - 4$

$\therefore \quad f'(c) = 2c - 4 = 0 \Rightarrow c = 2 \in (1, 3)$

Hence, the Rolle's theorem is verified.

15. Given, $f(x) = 3x^2 + 15x + 5 \Rightarrow \quad f'(x) = 6x + 15$

Also, $f(x + \Delta x) \approx f(x) + \Delta x\, f'(x)$

$\therefore \quad f(x+\Delta x) \approx 3x^2 + 15x + 5 + \Delta x(6x+15)$

Taking $x = 3$ and $\Delta x = 0.02$, we get

$f(3.02) \approx 3 \times 3^2 + 15 \times 3 + 5 + 0.02\,(6 \times 3 + 15) = 77 + 0.66$

$\Rightarrow \quad f(3.02) \approx 77.66$

OR

Here, $y = f(x) = [x(x - 2)]^2 = x^2(x - 2)^2$

$$\Rightarrow \quad \frac{dy}{dx} = 2x(x-2)^2 + 2x^2(x-2) = 4x(x-1)(x-2)$$

For y to be an increasing function, $\dfrac{dy}{dx} > 0$

$\Rightarrow \quad x(x - 1)(x - 2) > 0$

Case 1: When $-\infty < x < 0$

$\dfrac{dy}{dx} < 0 \Rightarrow y$ is a decreasing function.

Case 2: When $0 < x < 1$

$\dfrac{dy}{dx} > 0 \Rightarrow y$ is an increasing function.

Case 3: When $1 < x < 2$

$\dfrac{dy}{dx} < 0 \Rightarrow y$ is a decreasing function.

Case 4: When $2 < x < \infty$

$\dfrac{dy}{dx} > 0 \Rightarrow y$ is an increasing function.

$\therefore \quad y$ is an increasing function in $(0, 1) \cup (2, \infty)$

For tangent parallel to x-axis, $f'(x) = 0$

$\Rightarrow \quad 4x\,(x - 1)\,(x - 2) = 0 \Rightarrow x = 0, x = 1$ and $x = 2$

when $x = 0, y = 0$, when $x = 1, y = 1$, when $x = 2, y = 0$

The points on the curve at which the tangents are parallel to x-axis are $(0, 0)$ $(1, 1)$ and $(2, 0)$.

16. Let $I = \displaystyle\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{5/2(2x+4)-7}{\sqrt{x^2+4x+10}}dx$

$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{dx}{\sqrt{x^2+4x+10}}$$

$= I_1 + I_2$ (say) $\qquad$ ...(i)

where $I_1 = \dfrac{5}{2}\displaystyle\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$

Put $x^2 + 4x + 10 = t \Rightarrow (2x + 4)dx = dt$

$\therefore \; I_1 = \dfrac{5}{2}\displaystyle\int t^{-1/2}dt = 5\sqrt{t} = 5\sqrt{x^2+4x+10} + C_1 \qquad$ ...(ii)

and $I_2 = -7\displaystyle\int \frac{dx}{\sqrt{x^2+4x+10}} = -7\int \frac{dx}{\sqrt{(x+2)^2+(\sqrt{6})^2}}$

$= -7\log\left|x+2+\sqrt{x^2+4x+10}\right| + C_2 \qquad$ ...(iii)

From (i), (ii) and (iii), we get

$$I=5\sqrt{x^2+4x+10}-7\log|x+2+\sqrt{x^2+4x+10}|+C,$$

where $C = C_1 + C_2$.

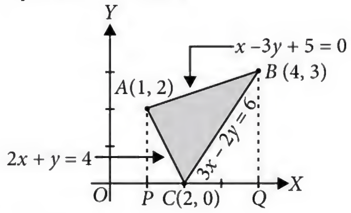
17. The given lines are $2x + y = 4$ ...(i)

$3x - 2y = 6$ ...(ii) and $x - 3y + 5 = 0$ ...(iii)

Solving (i) and (ii), we get $x = 2, y = 0$

Solving (ii) and (iii), we get $x = 4, y = 3$

Solving (i) and (iii), we get $x = 1, y = 2$



$\therefore$ Required area = area (ΔABC)

= area($ABQP$) – area (ΔAPC) – area (ΔBCQ)

$$=\int_1^4 \frac{x+5}{3}dx-\int_1^2(4-2x)dx-\int_2^4\frac{3x-6}{2}dx$$

$$=\frac{1}{3}\left[\frac{x^2}{2}+5x\right]_1^4-[4x-x^2]_1^2-\frac{1}{2}\left[\frac{3x^2}{2}-6x\right]_2^4$$

$$=\frac{1}{3}\left[28-\frac{1}{2}-5\right]-[4-3]-\frac{1}{2}[0+6]=\frac{7}{2}\text{ sq. units}$$

18. We have, $e^x\sqrt{1-y^2}\,dx+\frac{y}{x}dy=0$

or $\quad x\,e^x dx+\frac{y}{\sqrt{1-y^2}}dy=0$

Integrating both sides, we get

$$\Rightarrow x\cdot e^x-\int 1\cdot e^x dx-\frac{1}{2}\int\left(1-y^2\right)^{-1/2}(-2y)dy=C$$

$$\Rightarrow xe^x-e^x-\frac{1}{2}\frac{\left(1-y^2\right)^{1/2}}{1/2}=C\Rightarrow e^x(x-1)-\sqrt{1-y^2}=C$$

When $x = 0$ and $y = 1$, we have

$$e^0(0-1)-\sqrt{1-1}=C \Rightarrow C=-1$$

$\therefore \quad e^x(x-1)-\sqrt{1-y^2}=-1$, is the required solution.

OR

The equation of family of ellipses is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ...(i)

Differentiating both sides of (i) w.r.t. x, we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0 \Rightarrow \frac{y}{x}\left(\frac{dy}{dx}\right)=\frac{-b^2}{a^2} \quad \text{...(ii)}$$

Differentiating both sides of (ii) w.r.t. x, we get

$$\left(\frac{y}{x}\right)\left(\frac{d^2y}{dx^2}\right)+\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)\frac{dy}{dx}=0$$

$$\Rightarrow xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$, which is the required differential equation.

19. We have $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}+3\hat{k}$

Let $\vec{r}=\vec{a}+\vec{b}=2\hat{i}+3\hat{j}+4\hat{k}$ and $\vec{p}=\vec{a}-\vec{b}=-\hat{j}-2\hat{k}$

A unit vector perpendicular to both $\vec{r}$ and $\vec{p}$ is given as $\pm\frac{\vec{r}\times\vec{p}}{|\vec{r}\times\vec{p}|}$.

Now, $\vec{r}\times\vec{p}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}=-2\hat{i}+4\hat{j}-2\hat{k}$

So, the required unit vector

$$=\pm\frac{\left(-2\hat{i}+4\hat{j}-2\hat{k}\right)}{\sqrt{(-2)^2+4^2+(-2)^2}}=\mp\frac{\left(\hat{i}-2\hat{j}+\hat{k}\right)}{\sqrt{6}}.$$

20. Equation of plane passing through (0, 0, 0) is

$a(x) + b(y) + c(z) = 0$ ...(i)

It also passes through (3, –1, 2)

$\therefore \quad 3a - b + 2c = 0$ ...(ii)

Also, (i) is parallel to the line $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$

$\therefore \quad a - 4b + 7c = 0$ ...(iii)

Solving (ii) and (iii), we get

$$\frac{a}{-7+8}=\frac{b}{2-21}=\frac{c}{-12+1} \text{ or } \frac{a}{1}=\frac{b}{-19}=\frac{c}{-11}=\lambda \text{ (say)}$$

$\Rightarrow \quad a = \lambda, b = -19\lambda, c = -11\lambda$

Putting the values of a, b, c in (i), we get

$\lambda x - 19\lambda y - 11\lambda z = 0 \Rightarrow x - 19y - 11z = 0$

21. Suppose that the young man rides x km at 25 km per hour and y km at 40 km per hour. Then, the given problem can be formulated as

Maximize $Z = x + y$

Subject to the constraints, $x \geq 0, y \geq 0, 2x + 5y \leq 100,$

$\frac{x}{25}+\frac{y}{40}\leq 1 \Rightarrow 8x + 5y \leq 200$

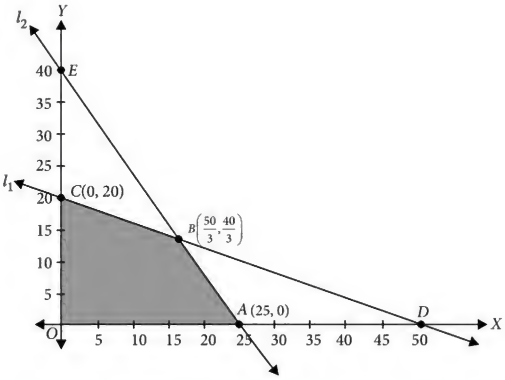
Now, we convert the system of the inequations into equations.

$l_1 : 2x + 5y = 100$ and $l_2 : 8x + 5y = 200$

Both the lines intersect at $B\left(\frac{50}{3},\frac{40}{3}\right)$

The solution set of the given system is the shaded region $OABC$.

The coordinates of corner points O, A, B, C are (0, 0), (25, 0), $\left(\frac{50}{3},\frac{40}{3}\right)$ and (0, 20) respectively.



Corner Points	Value of $Z = x + y$
$O(0, 0)$	0
$A(25, 0)$	25
$B\left(\frac{50}{3}, \frac{40}{3}\right)$	30 (Maximum)
$C(0, 20)$	20

So, $Z = x + y$ is maximum when $x = \frac{50}{3}$ and $y = \frac{40}{3}$.

Thus, the student can cover the maximum distance of 30 km, if he rides $\frac{50}{3}$ km at 25 km/hr and $\frac{40}{3}$ km at 40 km/hr.

The value indicated in this question is that maximum distance is covered in one hour with less pollution.

22. Suppose a man tosses a fair coin n times, we have $p = \frac{1}{2}$ and $q = 1 - \frac{1}{2} = \frac{1}{2}$

Then, $P(X = r) = {}^nC_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r}$

$= {}^nC_r \left(\frac{1}{2}\right)^n, r = 0, 1, 2, ..., n$

Given, $P(X \geq 1) > \frac{80}{100}$

$\Rightarrow \ 1 - P(X = 0) > \frac{80}{100} \Rightarrow P(X = 0) < 1 - \frac{80}{100}$

$\Rightarrow \ {}^nC_0 \left(\frac{1}{2}\right)^n < \frac{20}{100} \Rightarrow \left(\frac{1}{2}\right)^n < \frac{1}{5}$

Clearly $\frac{1}{2} \nless \frac{1}{5}, \left(\frac{1}{2}\right)^2 \nless \frac{1}{5}$ but $\left(\frac{1}{2}\right)^3 < \frac{1}{5}$

$\therefore \ \left(\frac{1}{2}\right)^n < \frac{1}{5} \Rightarrow n = 3, 4, 5, ...$

Thus, he must toss the coin atleast 3 times.

23. Let E_1 and E_2 be the events to select bag I and bag II respectively.

$\therefore \quad P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{2}$

Let E be the event that drawn ball is white.

$P(E/E_1) = \frac{4}{7}$ and $P(E/E_2) = \frac{3}{10}$

$\therefore \quad P$ (White ball was drawn from bag I)

$$= P(E_1/E) = \frac{P(E_1)P(E/E_1)}{P(E_1)P(E/E_1) + P(E_2)P(E/E_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{4}{7}}{\frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{3}{10}} = \frac{\frac{4}{7}}{\frac{4}{7} + \frac{3}{10}} = \frac{\frac{4}{7}}{\frac{40+21}{70}} = \frac{40}{61}$$

24. We have, by definition,

$$\int_a^b f(x)\,dx = \lim_{h \to 0} h\,[f(a) + f(a+h) + f(a+2h) + ... + f(a + (n-1)h)],$$

where $nh = b - a$

Here $a = 1$, $b = 3$, $f(x) = 2x^2 + 3$ and $nh = 2$

$$\therefore \int_1^3 (2x^2 + 3)dx = \lim_{h \to 0} h[\{2(1)^2 + 3\} + \{2(1+h)^2 + 3\} + ... + \{2\,(1 + (n-1)h)^2 + 3\}]$$

$$= \lim_{h \to 0} h \begin{bmatrix} 5n + 4h(1 + 2 + ... + (n-1)) \\ + 2h^2(1^2 + 2^2 + ... + (n-1)^2) \end{bmatrix}$$

$$= \lim_{h \to 0} [5nh + \frac{4nh(nh-h)}{2} + \frac{2nh(nh-h)(2nh-h)}{6}]$$

$$= 5 \times 2 + \frac{4 \times 2 \times 2}{2} + \frac{4 \times 2 \times 4}{6} = 10 + 8 + \frac{16}{3} = \frac{70}{3}$$

OR

Let $I = \int \frac{\sqrt{x^2+1}[\log(x^2+1) - 2\log x]}{x^4}\,dx$

$$= \int \frac{\sqrt{x^2+1}\,\log\left(\frac{x^2+1}{x^2}\right)}{x^4}\,dx$$

Put $\frac{x^2+1}{x^2} = t \Rightarrow x^2 = \frac{1}{t-1} \quad \Rightarrow \ dx = -\frac{1}{2x} \cdot \frac{1}{(t-1)^2}\,dt$

$$= -\frac{1}{2} \cdot \sqrt{t-1} \cdot \frac{1}{(t-1)^2}\,dt \ = -\frac{dt}{2(t-1)^{3/2}}$$

Also $\sqrt{x^2+1} = \sqrt{\frac{1}{t-1} + 1} = \sqrt{\frac{t}{t-1}}$

$$\therefore \quad I = \int \sqrt{\frac{t}{t-1}} \cdot \log t \cdot \frac{1}{1/(t-1)^2} \times \frac{-dt}{2(t-1)^{3/2}}$$

$$=-\frac{1}{2}\int\sqrt{t}\cdot\log t\,dt=-\frac{1}{2}\left[\frac{t^{3/2}}{3/2}\cdot\log t-\int\frac{t^{3/2}}{3/2}\cdot\frac{1}{t}dt\right]+C$$

$$=-\frac{1}{3}\left[t^{3/2}\log t-\int t^{1/2}dt\right]+C=\frac{-1}{3}\left[t^{3/2}\log t-\frac{2}{3}t^{3/2}\right]+C$$

$$=\frac{-1}{3}\left[\left(\frac{x^2+1}{x^2}\right)^{3/2}\log\left(\frac{x^2+1}{x^2}\right)-\frac{2}{3}\left(\frac{x^2+1}{x^2}\right)^{3/2}\right]+C$$

25. Consider $A=\begin{pmatrix}1&3&-2\\-3&0&-1\\2&1&0\end{pmatrix}$

We have $A = IA$

$$\begin{pmatrix}1&3&-2\\-3&0&-1\\2&1&0\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we get

$$\begin{pmatrix}1&3&-2\\0&9&-7\\0&-5&4\end{pmatrix}=\begin{pmatrix}1&0&0\\3&1&0\\-2&0&1\end{pmatrix}A$$

Applying $R_2 \to \frac{R_2}{9}$, we get

$$\begin{pmatrix}1&3&-2\\0&1&-7/9\\0&-5&4\end{pmatrix}=\begin{pmatrix}1&0&0\\1/3&1/9&0\\-2&0&1\end{pmatrix}A$$

Applying $R_3 \to R_3 + 5R_2$, we get

$$\begin{pmatrix}1&3&-2\\0&1&-7/9\\0&0&1/9\end{pmatrix}=\begin{pmatrix}1&0&0\\1/3&1/9&0\\-1/3&5/9&1\end{pmatrix}A$$

Applying $R_3 \to 9R_3$, we get

$$\begin{pmatrix}1&3&-2\\0&1&-7/9\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0\\1/3&1/9&0\\-3&5&9\end{pmatrix}A$$

Applying $R_1 \to R_1 - 3R_2$, we get

$$\begin{pmatrix}1&0&1/3\\0&1&-7/9\\0&0&1\end{pmatrix}=\begin{pmatrix}0&-1/3&0\\1/3&1/9&0\\-3&5&9\end{pmatrix}A$$

Applying $R_1 \to R_1-\frac{1}{3}R_3, R_2 \to R_2+\frac{7}{9}R_3$, we get

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}1&-2&-3\\-2&4&7\\-3&5&9\end{pmatrix}A$$

$$\therefore\quad A^{-1}=\begin{pmatrix}1&-2&-3\\-2&4&7\\-3&5&9\end{pmatrix}$$

OR

According to question, we have

$x + y + z = 12000$, $4x + 3y + 2z = 37000$,

$5x + 3y + 4z = 47000$

The system of equations can be written as $AX = B$

where, $A=\begin{bmatrix}1&1&1\\4&3&2\\5&3&4\end{bmatrix}; X=\begin{bmatrix}x\\y\\z\end{bmatrix}; B=\begin{bmatrix}12000\\37000\\47000\end{bmatrix}$

$$|A|=\begin{vmatrix}1&1&1\\4&3&2\\5&3&4\end{vmatrix}=1(12-6)-1(16-10)+1(12-15)$$
$$=-3\neq 0$$

$\therefore$ A^{-1} exists. So, system of equations has a unique solution and it is given by $X = A^{-1}B$

$$\therefore\quad \text{adj } A=\begin{bmatrix}6&-1&-1\\-6&-1&2\\-3&2&-1\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\cdot\text{adj } A=-\frac{1}{3}\begin{bmatrix}6&-1&-1\\-6&-1&2\\-3&2&-1\end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow\begin{bmatrix}x\\y\\z\end{bmatrix}=-\frac{1}{3}\begin{bmatrix}6&-1&-1\\-6&-1&2\\-3&2&-1\end{bmatrix}\begin{bmatrix}12000\\37000\\47000\end{bmatrix}=\begin{bmatrix}4000\\5000\\3000\end{bmatrix}$$

$\Rightarrow$ $x = 4000, y = 5000, z = 3000$

The values described in this question are resourcefulness, competence and determination.

26. We have, $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$

$$\Rightarrow \cos^{-1}\left[\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right]=\alpha$$

$$\Rightarrow \frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha \qquad \text{... (i)}$$

Squaring on both sides of (i), we get

$$\frac{x^2y^2}{a^2b^2}+\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)-\frac{2xy}{ab}\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}=\cos^2\alpha$$

$$\Rightarrow \frac{x^2y^2}{a^2b^2}+1-\frac{y^2}{b^2}-\frac{x^2}{a^2}+\frac{x^2y^2}{a^2b^2}-\frac{2xy}{ab}\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$$
$$=1-\sin^2\alpha$$

$$\Rightarrow \frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2xy}{ab}\left[\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right]=\sin^2\alpha$$

$$\Rightarrow \frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha \qquad \text{[From (i)]}$$

OR

We have $a * b = a + b + ab$ for all $a, b \in A$, where $A = R - \{-1\}$

Commutativity : Let $a, b \in R - \{-1\}$

We have, $a * b = a + b + ab = b + a + ba = b * a$

Hence, * is commutative.

Associativity : Let $a, b, c \in R - \{-1\}$

We have, $a * (b * c) = a * (b + c + bc)$

$= a + (b + c + bc) + a(b + c + bc)$

$= a + b + ab + c + (a + b + ab)c = (a + b + ab) * c = (a * b) * c$

Hence, * is associative.

Let $e \in A$ be the identity element. Then, $a * e = a = e * a$

$a * e = a + e + ae = a$ and $e * a = e + a + ea = a$

$\Rightarrow e(1 + a) = 0 \Rightarrow e = 0$ $\quad [\because a \neq -1]$

Hence, the identity element for * is $e = 0$.

Existence of inverse : Let $a \in R - \{-1\}$ and b be the inverse of a.

Then, $a * b = e = b * a$

$\Rightarrow a + b + ab = 0 = b + a + ba \Rightarrow b = -\dfrac{a}{a+1}$

Since, $a \in R - \{-1\}$

$\therefore a \neq -1 \Rightarrow a + 1 \neq 0 \Rightarrow b = \dfrac{-a}{a+1} \in R$

Also, if $-\dfrac{a}{a+1} = -1$

$\Rightarrow -a = -a - 1 \Rightarrow -1 = 0$, which is not possible.

Hence, $\dfrac{-a}{a+1} \in R - \{-1\}$

So, every element of $R - \{-1\}$ is invertible and the inverse of an element a is $\dfrac{-a}{a+1}$.

27. Let S, V, r and h be the surface area, volume, radius and height of the cylinder. Then

$S = \pi r^2 + 2\pi rh$ $\quad [\because$ Cylinder is open at the top$]$

$\Rightarrow h = \dfrac{S - \pi r^2}{2\pi r}$

Now, volume of cylinder, $V = \pi r^2 h$

$$\Rightarrow V = \pi r^2\left(\frac{S-\pi r^2}{2\pi r}\right) \qquad \left[\because h = \frac{S-\pi r^2}{2\pi r}\right]$$

$$\Rightarrow V = \frac{1}{2}(Sr - \pi r^3) \qquad \text{...(i)}$$

Differentiating (i) w.r.t. r, we get $\dfrac{dV}{dr} = \dfrac{1}{2}(S - 3\pi r^2)$

For maxima or minima, $\dfrac{dV}{dr} = 0 \Rightarrow S = 3\pi r^2$

Also, $\dfrac{d^2V}{dr^2} = \dfrac{-6\pi r}{2} < 0$

$\therefore$ Volume of cylinder is maximum, when $S = 3\pi r^2$

Now, $h = \dfrac{S - \pi r^2}{2\pi r} = \dfrac{3\pi r^2 - \pi r^2}{2\pi r} = r$

Hence, volume of cylinder is maximum, when $h = r$.

28. The equation of the plane passing through $(2, 5, -3)$ is $a(x - 2) + b(y - 5) + c(z + 3) = 0$ ...(i)

If plane (i) passes through the points $(-2, -3, 5)$ and $(5, 3, -3)$, then

$-4a - 8b + 8c = 0 \Rightarrow a + 2b - 2c = 0$ ...(ii)

and $3a - 2b + 0\cdot c = 0$ ...(iii)

Solving (ii) and (iii), we get

$$\frac{a}{-4}=\frac{b}{-6}=\frac{c}{-2-6} \Rightarrow \frac{a}{2}=\frac{b}{3}=\frac{c}{4}=\lambda\text{(say)}$$

$\Rightarrow a = 2\lambda, b = 3\lambda, c = 4\lambda$

Substituting the values of a, b, c in (i), we get

$2\lambda(x - 2) + 3\lambda(y - 5) + 4\lambda(z + 3) = 0$

$\Rightarrow 2x + 3y + 4z = 7$, which is the equation of the plane passing through the given three points.

Now the distance of $(7, 2, 4)$ from this plane is given by

$$\left|\frac{14+6+16-7}{\sqrt{4+9+16}}\right| = \frac{29}{\sqrt{29}} = \sqrt{29} \text{ units}$$

29. For $f(x)$ to be continuous at $x = 0$, we must have

$$\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(0)$$

Now, $\displaystyle\lim_{x\to0^-} f(x) = \lim_{h\to0}\frac{\sin(a+1)(0-h)+\sin(0-h)}{0-h}$

$$= \lim_{h\to0}\frac{-h\sin(a+1)-\sin h}{-h}$$

$$= \lim_{h\to0}\frac{\sin(a+1)h}{(a+1)h}\times(a+1)+\lim_{h\to0}\frac{\sin h}{h} = a+2 \qquad \text{...(i)}$$

And, $\displaystyle\lim_{x\to0^+} f(x) = \lim_{h\to0}\frac{\sqrt{0+h+b(0+h)^2}-\sqrt{0+h}}{b(0+h)^{3/2}}$

$$= \lim_{h\to0}\frac{\sqrt{h+bh^2}-\sqrt{h}}{bh^{3/2}} = \lim_{h\to0}\frac{(\sqrt{1+bh}-1)\sqrt{h}}{bh^{3/2}}$$

$$= \lim_{h\to0}\frac{\sqrt{1+bh}-1}{bh}\times\frac{\sqrt{1+bh}+1}{\sqrt{1+bh}+1}$$

$$= \lim_{h\to0}\frac{1+bh-1}{bh(\sqrt{1+bh}+1)} = \lim_{h\to0}\frac{bh}{bh(\sqrt{1+bh}+1)} = \frac{1}{2} \qquad \text{...(ii)}$$

Also, $f(0) = c$ ...(iii)

$\therefore$ From (i), (ii) and (iii), we get

$$a+2=\frac{1}{2}=c \Rightarrow a=\frac{-3}{2} \text{ and } c=\frac{1}{2}$$

and b can be any real number.

MPP | MONTHLY Practice Problems

Class XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. Let A be any 3×3 invertible matrix. Then which one of the following is not always true?
   (a) adj (adj (A)) $= |A| \cdot (\text{adj}(A))^{-1}$
   (b) adj (adj (A)) $= |A|^2 \cdot (\text{adj}(A))^{-1}$
   (c) adj $(A) = |A| \cdot A^{-1}$
   (d) adj (adj (A)) $= |A| \cdot A$

2. The period of $f(x) = |\sin x| + |\cos x|$ is
   (a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

3. Let a_n, $n = 1, 2, 3, 4$ represent four distinct positive real numbers other than unit such that each pair of the logarithm of a_n and the reciprocal of logarithm denotes a point on a circle, whose centre lies on y-axis. Then the product of these four number is
   (a) 0 (b) 1 (c) 2 (d) 3

4. The area of the figure bounded by the parabolas $x = -2y^2$ and $x = 1 - 3y^2$ is
   (a) $\frac{4}{3}$ square units (b) $\frac{2}{3}$ square units
   (c) $\frac{3}{7}$ square units (d) $\frac{6}{7}$ square units

5. Let $g(x) = \log f(x)$ where $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x + 1) = x f(x)$. Then, for $N = 1, 2, 3, \ldots$,
   $$g''\left(N+\frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$
   (a) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}$
   (b) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}$
   (c) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$
   (d) $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$

6. $\int (\sec x)^m (\tan^3 x + \tan x)\, dx$ is equal to
   (a) $\sec^{m+2} x + C$ (b) $\tan^{m+2} x + C$
   (c) $\frac{\sec^{m+2} x}{m+2} + C$ (d) $\frac{\tan^{m+2} x}{m+2} + C$

One or More Than One Option(s) Correct Type

7. Let E and F be two independent events. The probability that both E and F happen is $\frac{1}{12}$ and the probability that neither E nor F happens is $\frac{1}{2}$. Then
   (a) $P(E) = \frac{1}{3}, P(F) = \frac{1}{4}$ (b) $P(E) = \frac{1}{2}, P(F) = \frac{1}{6}$
   (c) $P(E) = \frac{1}{6}, P(F) = \frac{1}{2}$ (d) $P(E) = \frac{1}{4}, P(F) = \frac{1}{3}$

8. The direction cosines of two lines are connected by relations $l + m + n = 0$ and $4l$ is the harmonic mean between m and n. Then,
   (a) $\frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2} = -3/2$
   (b) $l_1 l_2 + m_1 m_2 + n_1 n_2 = -1/2$
   (c) $l_1 m_1 n_1 + l_2 m_2 n_2 = -\sqrt{6}/9$
   (d) $(l_1 + l_2)(m_1 + m_2)(n_1 + n_2) = \frac{\sqrt{6}}{18}$

9. For 3×3 matrices M and N, which of the following statement(s) is (are) not correct?
   (a) $N^T M N$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric.

(b) $MN - NM$ is skew symmetric for all symmetric matrices M and N.

(c) MN is symmetric for all symmetric matrices M and N.

(d) (adj M) (adj N) = adj (MN) for all invertible matrices M and N.

10. Let A, B and C be 2×2 matrices with entries from the set of real numbers. Define * as follows:

$A * B = \frac{1}{2}(AB' + A'B)$, then

(a) $A * B = B * A$ (b) $A * A = AA'$

(c) $A * (B + C) = A * B + A * C$

(d) $A * I = \frac{1}{2}(A + A')$

11. The difference of the tangents of the angles which the lines $(\tan^2\alpha + \cos^2\alpha)\, x^2 - 2xy \tan\alpha + \sin^2 a y^2 = 0$ make with the axis of x is

(a) 2 (b) 3 (c) −2 (d) −3

12. Let $f: R \to R$ be defined as $f(x) = \dfrac{\sin \pi\{x\}}{x^2 - x + 1}$ $\forall x \in R$, where $\{x\}$ is fractional part function. Then function

(a) f is neither even nor odd.

(b) f is a zero function.

(c) f is many-one and non-constant function.

(d) f is one-one function.

13. Let $\vec{a} = x\hat{i} + x^2\hat{j} + 2\hat{k}$, $\vec{b} = -3\hat{i} + \hat{j} + \hat{k}$, $\vec{c} = (3x + 11)\hat{i} + (x - 9)\hat{j} - 3\hat{k}$ be three vectors. Then, angle between $\vec{a}$ and $\vec{b}$ is acute angle and between $\vec{c}$ and $\vec{a}$ is obtuse, if x lies in

(a) $(-\infty, 1) \cup (2, 3)$ (b) $(-\infty, 1)$

(c) (2. 3) (d) None of these

Comprehension Type

Given that for each $a \in (0, 1)$, $\lim\limits_{h \to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$ exists. Let this limit be $g(a)$. In addition, it is given that the function $g(a)$ is differentiable on (0, 1).

14. The value of $g\left(\frac{1}{2}\right)$ is

(a) π (b) 2π (c) $\pi/2$ (d) $\pi/4$

15. The value of $g'\left(\frac{1}{2}\right)$ is

(a) $\pi/2$ (b) π (c) $-\pi/2$ (d) 0

Matrix Match Type

16. Match the following.

	Column I		Column II
P.	The number of independent constants in the equation of a straight line in 3 dimensions is	1.	−9
Q.	The square of distance of the point (1, 2, 3) from the line $x^2 + y^2 = 0$ is	2.	9
R.	If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ are coplanar, then $2k$ is	3.	5
S.	If the image of the point $P(3, 5, 7)$ in the plane $2x + y + z = 0$ is (a, b, c), then $a + b + c$ is	4.	4

	P	Q	R	S
(a)	4	1	2	3
(b)	3	2	1	4
(c)	4	3	2	1
(d)	2	1	4	3

Integer Answer Type

17. For any differentiable function $y = f(x)$, the value of $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 \dfrac{d^2x}{dy^2}$ is

18. Let $f(x)$ be a polynomial of degree 4 such that $f(1) = 1$, $f'(1) = 2$, $f''(1) = 4$, $f'''(1) = 12$ and $f^4(1) = 24$. Then $f(2)$ is

19. If the line $\dfrac{x-3}{2} = \dfrac{y+2}{-1} = \dfrac{x+4}{3}$, lies in the plane $lx + my - z = 9$, then $l^2 + m^2$ is equal to

20. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \dfrac{3\pi}{2}$, then

$$\sum \frac{(x^{101} + y^{101})(x^{202} + y^{202})}{(x^{303} + y^{303})(x^{404} + y^{404})} =$$

◇◇

Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......

No. of questions correct ......

Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.